The Complex Numbers
Section 1.5

Real Numbers

Represented by a number line

Divided into rational numbers and irrational numbers

Complex Numbers

Have form a + bi

a = real part

b = imaginary part

i = imaginary unit

$i = \sqrt{-1}$

$i^2 = -1$

Pure Imaginary Numbers

When a = 0 (no real part) Ex: 8i or $i\sqrt{2}$

Examples

Rewrite as a complex number.

1) $\sqrt{-36}$

$\sqrt{-1} \cdot \sqrt{36}$

$i \cdot 6$

$6i$

2) $\sqrt{-15}$

$\sqrt{-1} \cdot \sqrt{15}$

$i\sqrt{15}$

3) $\sqrt{-3}\sqrt{-6}$

$i\sqrt{3} \cdot i\sqrt{6}$

$i^2\sqrt{18}$

$-1 \cdot \sqrt{9} \cdot \sqrt{2}$

$-1 \cdot 3\sqrt{2}$

$= -3\sqrt{2}$

Simplify.

4) $(6 - 5i) + (9 + 7i) =$

$15 + 2i$

Conjugates

6) $(8 - 5i)(8 + 5i) =$

FOIL

$64 + 40i - 40i - 25i^2 = 64 + 25 = 89$

5) $(7 + 3i)(4 - i) =$

FOIL

$28 - 7i + 12i - 3i^2 = 28+5i+3 = 31 + 5i$

7) $(8 - 5i) + (8 + 5i)$

16

Conjugates

-In the form a + bi and a - bi

-Sum is a real number

-Product is nonnegative real number

Simplify to a+bi form.

8) $\frac{1}{3-4i}\ \frac{(3+4i)}{(3+4i)} = \frac{3+4i}{9 - 16i^2} = \frac{3+4i}{9+16}$

$= \frac{3+4i}{25} = \frac{3}{25} + \frac{4}{25}i$

9) $\frac{8}{i} \cdot \frac{-i}{-i} = \frac{-8i}{-i^2} = \frac{-8i}{-(-1)}$

$= \frac{-8i}{1} = -8i$

10) $\frac{5+2i}{3+i}\ \frac{(3 - i)}{(3 - i)} = \frac{15 - 5i + 6i - 2i^2}{9 - 3i + 3i - i^2}$

$= \frac{15 + i + 2}{9 - (-1)} = \frac{17 + i}{10} = \frac{17}{10} + \frac{1}{10}i$

11) $i^2 = -1$

$i^3 = i^2 \cdot i = -1 \cdot i = -i$

$i^4 = i^3 \cdot i = -i \cdot i = -i^2 = 1$

$i^5 = 1 \cdot i = i$

$i^6 = i \cdot i = i^2 = -1$

$i^7 = i^6 \cdot i = -1 \cdot i = -i$

12) $i^{-4} = \frac{1}{i^4} = \frac{1}{1} = 1$

Equal Complex Numbers
a+bi and c+di are equal if...
a = c and b = d

Example 13
Find x and y. $2x + y + (3-5x)i = 1 - 7i$

$2x + y = 1$ (can't solve with 2 variables)

$3 - 5x = -7$ (solve for x first)

$-3 \quad -3$

$-5x = -10$

$x = 2$

2nd plug in x= 2 for other equation and solve for y

$2(2) + y = 1$

$4 + y = 1$

$-4 \quad -4$

$y = -3$

Homework

p28 #1-29 odd