

## Section 4.2 Notes: Operations on Functions

Given  $f$  and  $g$  are functions...

1. **Sum** of  $f$  and  $g$ :  $(f + g)(x) = f(x) + g(x)$

2. **Difference** of  $f$  and  $g$ :  $(f - g)(x) = f(x) - g(x)$

3. **Product** of  $f$  and  $g$ :  $(f \cdot g)(x) = f(x) \cdot g(x)$

4. **Quotient** of  $f$  and  $g$ :  $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ , provided  $g(x) \neq 0$

**Example 1**

Given  $f(x) = x^2$  and  $g(x) = x + 1$

**SUM**

$$(f + g)(x) = (x^2) + (x + 1) = x^2 + x + 1$$

**DIFFERENCE**

$$(f - g)(x) = (x^2) - (x + 1) = x^2 - x - 1$$

**PRODUCT**

$$(f \cdot g)(x) = (x^2)(x + 1) = x^3 + x^2$$

### The Composite Function

- Given functions  $f$  and  $g$ , denoted  $(f \circ g)(x) = f(g(x))$
- Read as "f of g" (the open circle is NOT a multiplication symbol)
- $x$  is in the domain of function  $g$  and  $g(x)$  is in the domain of function  $f$

#### Example 2

Given  $f(x) = x^2$  and  $g(x) = x + 1$ , find  $(f \circ g)(x)$ .

$$f(g(x)) = f(x+1) = (x+1)^2 = (x+1)(x+1) = x^2 + x + x + 1$$

$x^2 + 2x + 1$

#### Example 3

Using the same equations above, find  $(g \circ f)(x)$ .

$$g(x^2) = (x^2) + 1$$

$x^2 + 1$

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Given  $f(x) = x^2 + x$  and  $g(x) = x + 1$

5.  $(f + g)(x)$

$$(x^2 + x) + (x + 1)$$

$$x^2 + 2x + 1$$

6.  $(f - g)(x)$

$$(x^2 + x) - (x + 1)$$

$$x^2 + x - x - 1 = x^2 - 1$$

7.  $(f \cdot g)(x)$

$$(x^2 + x)(x + 1)$$

$$x^3 + x^2 + x^2 + x = x^3 + 2x^2 + x$$

8.  $(f \div g)(x)$

$x + 1 = 0$   
 $x = -1$

$$\frac{x^2 + x}{x + 1} = \frac{x \cdot (x + 1)}{x + 1} = x \quad (x \neq -1)$$

9. a)

$$f(g(2)) = f(2 + 1) = f(3)$$

$$3^2 + 3 = 12$$

b)

$$(f \circ g)(x) = f(g(x)) = f(x + 1)$$

$$(x + 1)^2 + (x + 1)$$

$$= x^2 + 2x + 1 + x + 1$$

10. a)

$$(g \circ f)(2) = g(f(2)) = g(2^2 + 2) = g(6)$$

$$x^2 + 3x + 2$$

$$g(6) = 6^2 + 3(6) + 2 = 36 + 18 + 2 = 56$$

$$(g \circ f)(x) = g(f(x)) = g(x^2 + x)$$

$$(x^2 + x) + 1 = x^2 + x + 1$$

$$6 + 1 = 7$$