Section 4.3 Notes

1) Reflection over the x-axis

- $y = -f(x)$ is a reflection of $y = f(x)$ over x-axis
- $(x, -y)$ is the reflected coordinate of (x, y)

Examples

$y = x^2 - 4$ OR $y = 2x + 1$

$y = -(x^2 - 4) = -x^2 + 4$ $y = -(2x + 1) = -2x - 1$

2) Entire graph above the x-axis

- Absolute Value

Example

$y = x^2 - 5$ becomes $y = |x^2 - 5|$

3) Reflection over the y-axis

- $y = f(-x)$ is a reflection of $y = f(x)$ over the y-axis
- $(-x, y)$ is the reflected coordinate of (x, y)

Examples

$y = 2^x$ OR $y = (x + 2)^2$

$y = 2^{-x}$ $y = (-x + 2)^2$

4) Reflection over the line y = x

- Change the x and y
- (y, x) is the reflected coordinate of (x, y)

Examples

$y = 3x + 1$ OR $y = x^3$

$x = 3y + 1 \longrightarrow y = \frac{x-1}{3}$ $x = y^3 \longrightarrow y = \sqrt[3]{x}$

$-1 \quad -1$

$\frac{x-1}{3} = \frac{3y}{3}$

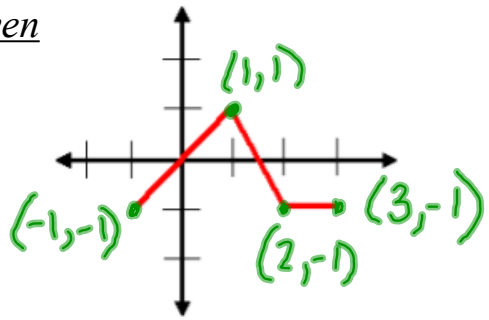
$y = \frac{1}{3}x - \frac{1}{3}$

* cube root both sides

Example 1

Given the graph of y = f(x) sketch the graph of…

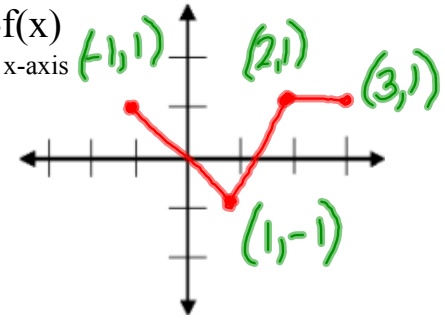
Given



a) y = -f(x)

reflection over x-axis

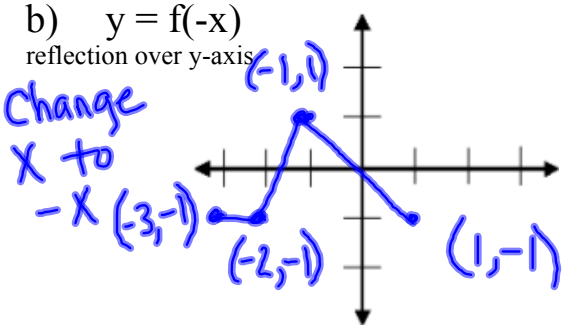
Change y to -y



b) y = f(-x)

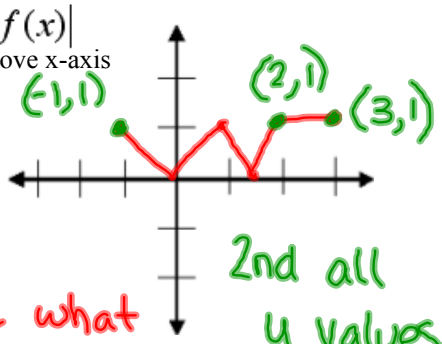
reflection over y-axis

Change x to -x



c) $y = |f(x)|$

everything above x-axis



* 1st Trace what is already above the x-axis

2nd all y values become positive

Example 2

Sketch the graph of the equation and the reflection of the graph over y = x. Give the equation of the new graph.

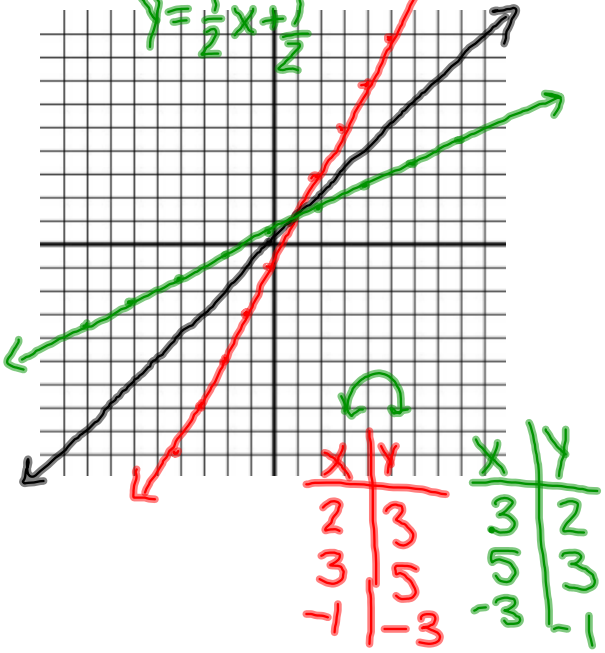
a) $y = 2x - 1$

Change x/y

$x = 2y - 1$

$x + 1 = 2y$

$y = \frac{1}{2}x + \frac{1}{2}$



x	y
2	3
3	5
-1	-3

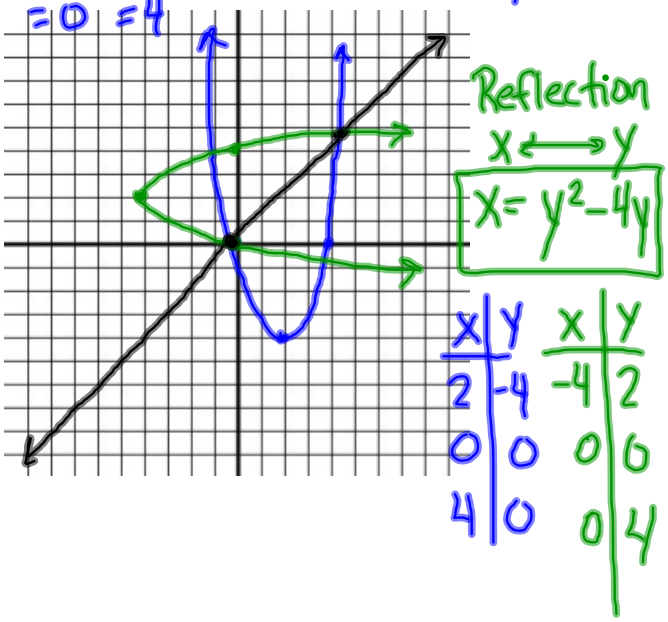
x	y
3	2
5	3
-3	-1

b) $y = x^2 - 4x$

$\frac{-b}{2a} = \frac{4}{2} = 2$ → plug in

$x(x-4) = 0$ $x = 4$

(2, -4)



Reflection

$x \longleftrightarrow y$

$x = y^2 - 4y$

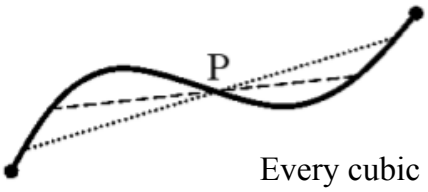
x	y
2	-4
0	0
4	0

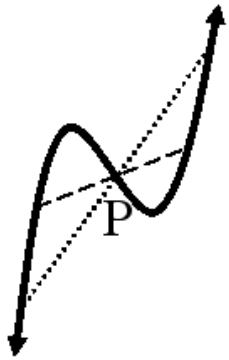
x	y
-4	2
0	0
0	4

Point of Symmetry (POS)

A point P is called POS of a graph if you can pair the points so P is the midpoint of the segment joining each pair of points.

Examples





Every cubic equation has a POS

$$x = \frac{-b}{3a}$$

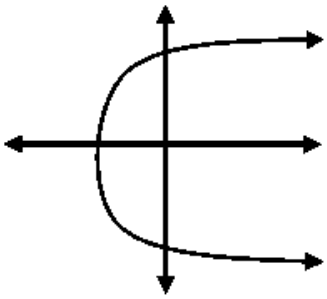
Axis of Symmetry (AOS) or Line of Symmetry (LOS)

- Splits graph in half
- Quadratic Equations AOS is $x = \frac{-b}{2a}$

How to Test for Symmetry of a Graph

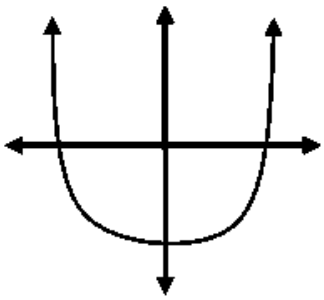
Symmetric to the x-axis

- (x, y) and (x, -y) are on the graph
- Test: leave x alone and substitute -y in for y
- Do you get an equivalent equation?



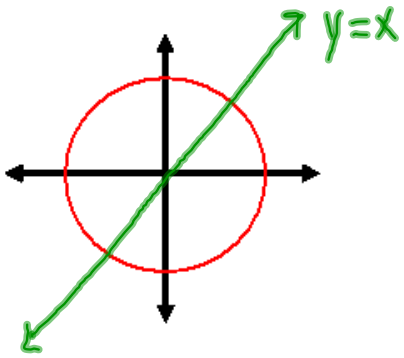
Symmetric to the y-axis

- (x, y) and (-x, y) are on the graph
- Test: leave y alone and substitute -x in for x
- Do you get an equivalent equation?



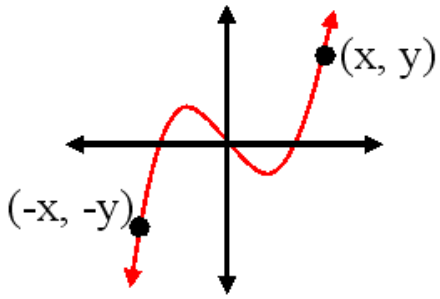
Symmetric to the line y = x

- (x, y) and (y, x) are on the graph
- Test: Interchange x and y
- Do you get an equivalent equation?



Symmetric to the origin

- (x, y) and (-x, -y) are on the graph
- Test: Substitute -x for x and -y for y
- Do you get an equivalent equation?



Examples

Test for symmetry with…

a) the x-axis (y becomes -y) b) the y-axis (x becomes -x) c) line y = x (interchange x & y) d) the origin (x becomes -x, y becomes -y)

$(-x)^{even} = pos$

$(-x)^{odd} = Neg.$

#3

$x^4 + y^4 = 1$

a) $x^4 + (-y)^4 = 1$

$x^4 + y^4 = 1$ (yes)

b) $(-x)^4 + y^4 = 1$

$x^4 + y^4 = 1$ (yes)

c) $y^4 + x^4 = 1$ (yes)

d) $(-x)^4 + (-y)^4 = 1$

$x^4 + y^4 = 1$ (yes)

#4

$xy^3 = 1$

a) $x(-y)^3 = 1$

$-xy^3 = 1$ (no)

b) $(-x)y^3 = 1$

$-xy^3 = 1$ (no)

c) $yx^3 = 1$ (no)

d) $(-x)(-y)^3 = 1$

$+(+)y^3 = 1$ (yes)

#5 $x^2 + xy = 1$

$x(x + y) = 1$

a) $x^2 + x(-y) = 1$

$x^2 - xy = 1$ (no)

b) $(-x)^2 + (-x)y = 1$

$x^2 - xy = 1$ (no)

c) $y^2 + yx = 1$ (no)

d) $(-x)^2 + (-x)(-y) = 1$

$x^2 + xy = 1$ (yes)

#6 $-x^2 + y = 7$

$y = x^2 + 7$

a) $-y = x^2 + 7$ (no)

$y = -x^2 - 7$

b) $y = (-x)^2 + 7$

$y = x^2 + 7$ (yes)

c) $x = y^2 + 7$

$\sqrt{y^2} = \sqrt{x - 7}$ (no)

$y = \pm\sqrt{x - 7}$

d) $-y = (-x)^2 + 7$

$-y = x^2 + 7$ (no)

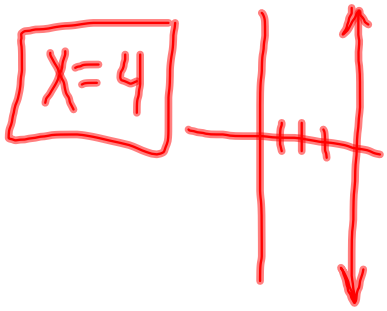
Example 7

Find the AOS for

$y = x^2 - 8x - 7$

$x = \frac{-b}{2a} = \frac{8}{2(1)} = \frac{8}{2}$

$x = 4$



Example 8

Find the POS for

$y = x^3 - 6x^2 + 5x + 7$

a b c d

$x = \frac{-b}{3a} = \frac{6}{3(1)} = \frac{6}{3} = 2$

$2^3 - 6(2)^2 + 5(2) + 7$

$8 - 24 + 10 + 7 = 1$

$(2, 1)$

Homework

p136 #1 - 13 odd mint

p136 #15, 21, 23, 25 vanilla