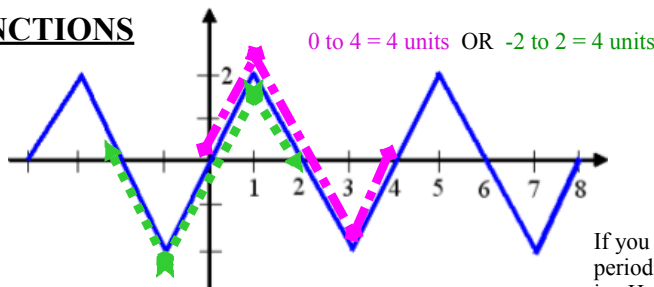


PERIODIC FUNCTIONS



If you do something periodically you repeat it. i.e. Hopefully like brushing your teeth :)

This graph is periodic; it has a positive period P .

Fundamental Period is the smallest period of a function.

*The Period = $P=4$ for the graph above because the graph repeats every 4 units.

A function f is periodic if
 $f(x) = f(x + mp)$

p = value of fundamental period
 m = number of periods within a value

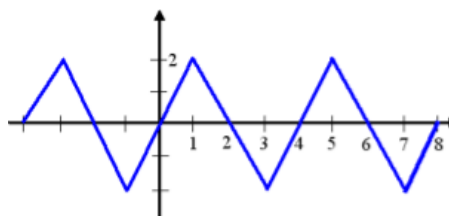
This is saying that the range $f(x) = y$ will repeat itself as you add or subtract multiple of the period.

Example:
 $f(-1) = -2$
 $f(-1 + 4) = f(3) = -2$ or $f(-1 + 2(4)) = f(7) = -2$

Example 1

Find the fundamental period of $f(x)$.

Ask yourself...
How long does one cycle take?



$P=4$

Example 2

Find $f(103)$

$\frac{103}{4} = 25 \frac{3}{4}$ ← Remainder

$f(103) = f(3) = -2$

Example 3

Find $f(106)$

$\frac{106}{4} = 26 \frac{2}{4}$ ← Rem.

$f(106) = f(2) = 0$

* If there is no remainder then $f(0)$

Alg III 4.4 lesson

AMPLITUDE is the average of the maximum and minimum values of a periodic function.

$$A = \frac{M - m}{2}$$

M = maximum value

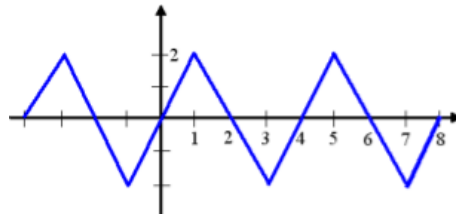
m = minimum value

Example 4

Find the amplitude of $f(x)$

$$\text{Max} = 2 \quad \text{min} = -2$$

$$A = \frac{2 - (-2)}{2} = \frac{4}{2} = 2$$



Stretching and Shrinking a graph

Vertical stretch or shrink:

$$y = c f(x)$$

$$y = 2f(x)$$

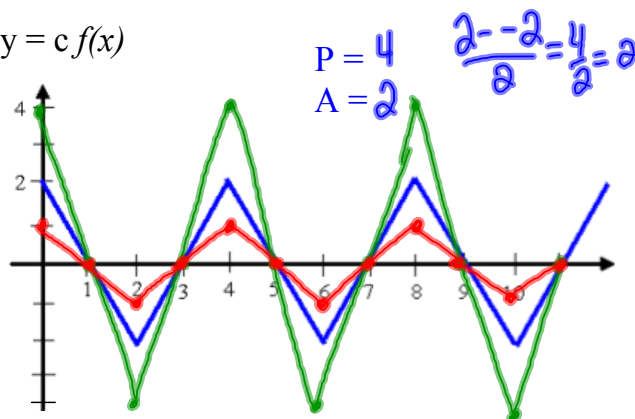
$$P = 4$$

$$A = \frac{4 - (-4)}{2} = 4$$

$$y = 1/2 f(x)$$

$$P = 4$$

$$A = \frac{1 - (-1)}{2} = 1$$



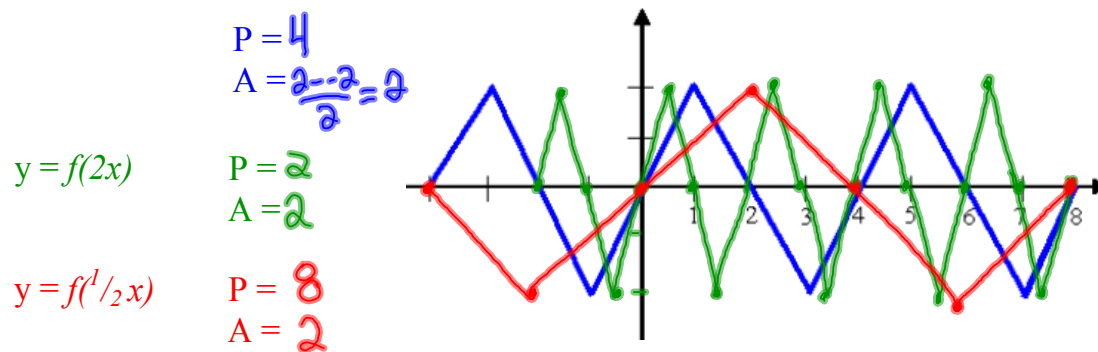
If $|c| > 1$ the graph is vertically **STRETCHED**

If $0 < |c| < 1$ the graph is vertically **SHRUNK**

Note: changes amplitude, NOT period

Alg III 4.4 lesson

Horizontal stretch or shrink: $y = f(cx)$



If $|c| > 1$ the graph is horizontally **Shrunk**

If $0 < |c| < 1$ the graph is horizontally **Stretched**

Note: changes period, NOT amplitude

Changing Period and Amplitude of a Periodic Function

If a periodic function f has a period P and amplitude A ...

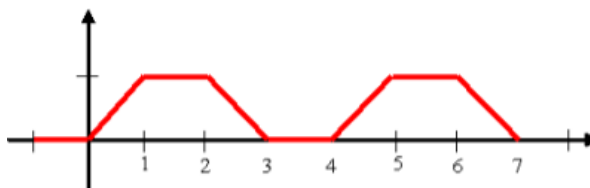
Then $y = cf(x)$ has period **P** and amplitude **cA** .

Then $y = f(cx)$ has period **P/c** and amplitude **A** .

Example 5

Find the period and amplitude of $f(x)$

$P = 4$ $A = \frac{1-0}{2}$
 $= \frac{1}{2}$



Example 6

Find the period and amplitude of $y = 2 f(x)$

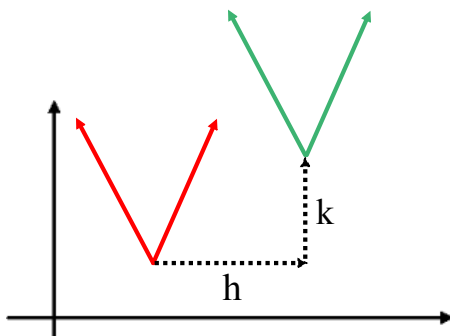
$P = 4$
 $A = 2(\frac{1}{2}) = 1$
 c-value original amplitude

Example 7

Find the period and amplitude of $y = f(\frac{1}{2} x)$

$P = \frac{4}{\frac{1}{2}} = 8$ ← original period
 $c = \frac{1}{2}$
 $A = \frac{1}{2}$

TRANSLATING A GRAPH



$y - k = f(x - h)$ is the same as

$y = f(x - h) + k$

horizontal movement

if $(x-h)$
 * move h units right

if $(x+h)$
 * move h units left

vertical movement

if $(-k) \downarrow$

if $(+k) \uparrow$

Example 8Graph $y = |x|$

$$y = |x - 2| + 0 \quad (2, 0)$$

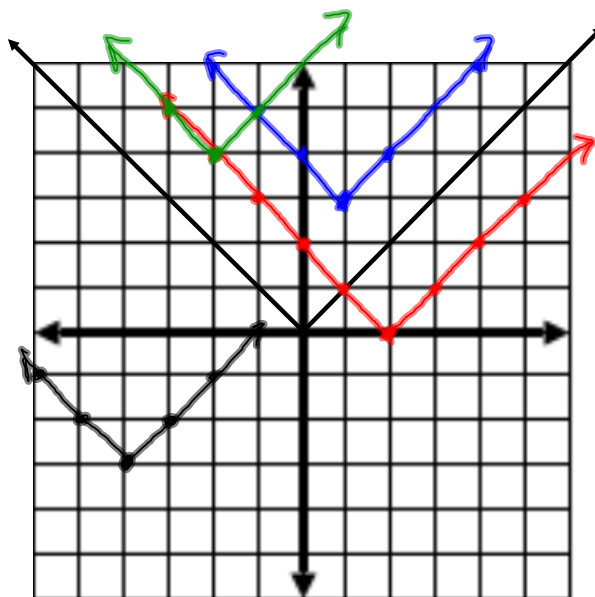
→ right 2

$$\begin{array}{l} \text{right} \\ \text{3 up} \end{array} y = |x - 1| + 3 \quad (1, 3)$$

$$y = |x + 2| + 4 \quad (-2, 4)$$

$$y = |x + 4| - 3 \quad \begin{array}{l} 2 \text{ left} \\ 4 \text{ up} \end{array}$$

$$\begin{array}{l} 4 \text{ left} \\ 3 \text{ Down} \end{array} \quad (-4, -3)$$



Note:

When doing multiple translations think in terms of order of operations

Reminder:

$$(x - h)^2 + (y - k)^2 = r^2$$

circle with center (h, k) and radius $= r$ **HOMEWORK**

p143 #1 - 4

#5, 7, 9abc, 10, 11, 12a