

*Section 4.5*INVERSES

- Given a function $f(x)$ the inverse is $f^{-1}(x)$
(the -1 is NOT a power!)
- Find the inverse by...
 - 1) Interchange x and y
 - 2) Solve for y
 - 3) Write the inverse notation $f^{-1}(x)$

Example 1Find the inverse of $f(x) = 3x - 1$

1) First interchange x and y $x = 3y - 1$

$+1 \quad +1$

2) Solve for y

$$\frac{x+1}{3} = \frac{3y}{3} \quad \text{then} \quad y = \frac{x+1}{3}$$

$$y = \frac{1}{3}x + \frac{1}{3}$$

3) Write using the inverse notation

$$f^{-1}(x) = \frac{1}{3}x + \frac{1}{3}$$

Example 1 (continued)

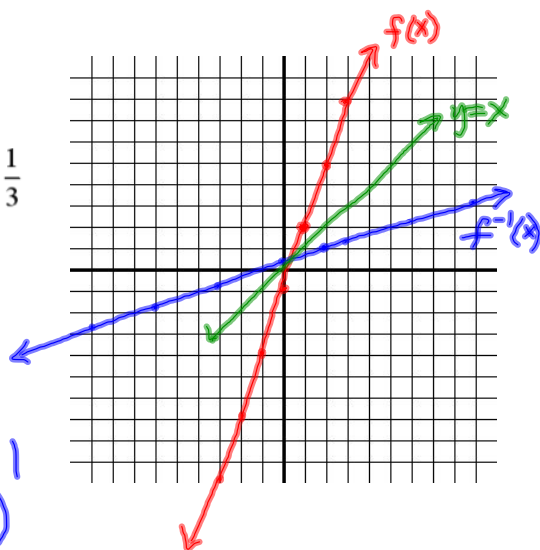
Graph $f(x)$ and $f^{-1}(x)$

$$f(x) = 3x - 1 \qquad f^{-1}(x) = \frac{1}{3}x + \frac{1}{3}$$

$(1, 2)$	$(2, 1)$
$(0, -1)$	$(-1, 0)$
$(-1, -4)$	$(-4, -1)$

Find $f(1) = 2$ $f^{-1}(2) = 1$
 $(1, 2)$ $(2, 1)$

Reflecting over the $y = x$ line
 Interchange x and y of $f(x)$ to get coordinates of $f^{-1}(x)$



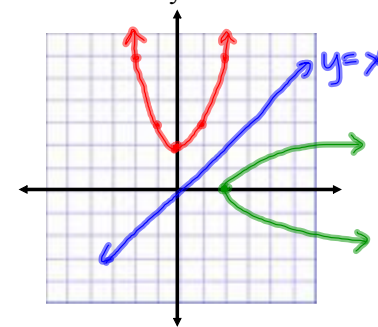
Example 2

Graph $f(x) = x^2 + 2$ and its reflection over the line $y = x$

Find the $f^{-1}(x)$

$f(x)$	
x	y
1	3
-1	3
2	6
-2	6
0	2

$f^{-1}(x)$	
x	y
3	1
3	-1
6	2
6	-2
2	0



The inverse does not exist (DNE) because the reflection fails the VLT. Thus, $x = y^2 + 2$ is not a function.

One - To - One Functions

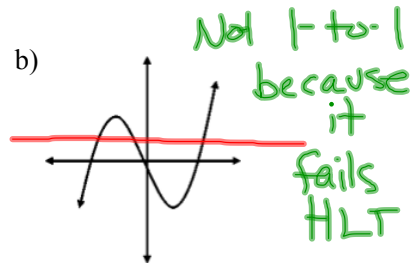
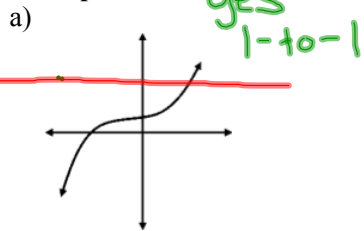
A function that has an inverse.
(The given equation is a function and the inverse is a function.)

HOW TO CHECK IF A FUNCTION IS 1-to-1

$f(x)$ is a function if it passes the VLT (vertical) but...

- $f^{-1}(x)$ is NOT a function if $f(x)$ fails the HLT (horizontal)
- $f^{-1}(x)$ is a function if $f(x)$ passes the HLT

Example 3



Example 4

Suppose $f(x)$ has an inverse.

If $f(0) = -1$ and $f(-1) = 2$

find...

$f(x)$	$f^{-1}(x)$
$(0, -1)$	$(-1, 0)$
$(-1, 2)$	$(2, -1)$

a) $f^{-1}(-1)$

$= \boxed{0}$

b) $f^{-1}(f(0))$

$f^{-1}(-1) = \boxed{0}$

c) $f(f^{-1}(-1))$

$f(0) = \boxed{-1}$

FACT...

$f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$

You get back what you plug in... f and f^{-1} "cancel each other"

