Algebra 1: ***Hutschenreuter*** **Name _______________________________**
Chapter 10 Notes

10.1 "Adding and Subtracting Polynomials"

Polynomial- an expression where terms are being either added and/or subtracted together
Ex: $6x^4 + 3x^3 + 5x^2 + 7x + 3$

Standard Form of a Polynomial- the terms are placed in descending order, from largest exponent to smallest exponent. Ex: $2x^2 + x + 3x^3 + 9$ → _______________________

Degree of Each Term- the exponent of the variable
Ex: $5x^3$… ___ is the degree

Degree of a Polynomial- the largest degree of its terms
Ex: $9x^4 + 5x^3 + x^2 + x + 7$…. ___ is the degree of the polynomial

Leading Coefficient- the coefficient of the first term of a polynomial written in standard form
Ex: $7x^3 + 2x^2 + 5x + 8$…. ___ is the leading coefficient

Classification by Number of Terms:

Monomial- a polynomial with only 1 term Ex: $3x$ or 6

Binomial- A polynomial with 2 terms Ex: $3x + 4$ or $2x^2 + 5x$

Trinomial- a polynomial with three terms Ex: $2x^2 + 4x + 1$

- Beyond trinomial, we just use the generic word "polynomial" to classify…

Classification by Degree:

0: constant Ex: 5
1: linear Ex: $5x + 2$
2: quadratic Ex: $5x^2 - 4$
3: cubic Ex: $5x^3 + 3x^2 - x - 1$
4: quartic Ex: $5x^4 + x - 7$

Polynomial	Degree	Classified by Degree	Classified by # of Terms
3			
$6x$			
$6x + 2$			
$3x^2 + 6x + 2$			
x^3			
$x^3 + 2x^2 +3x + 1$			
$x^4 + 2x^2 + -x - 1$			

Ex 1: Adding Polynomials
$(3x^2 - 2x + 4) + (4x^2 + 7x - 5) + (2x^2 - 3)$

Horizontal Adding Vertical Adding

Ex 2: Subtracting Polynomials

*You have to distribute the negative sign to all terms in the parentheses!

$(-2x^3 + 5x^2 – x + 8) – (–2x^3 + 3x – 4)$

Ex 3:

$(9x^3 + 12) + (6x^2 - 4x + 5)$

Ex 4:

$(x^3 - x) - (x^2 + 5x - 2)$

10.2 "Multiplying Polynomials"

Remember the distributive property?

Ex 1: $6x(x^2 + 3x + 6) =$

This is multiplying a monomial by a polynomial

What if we have 2 Binomials?

Ex 2: $(2x + 1)(x – 5)$

We have to do the distributive property TWICE!

$2x(x – 5) +1(x – 5) =$

In math, we call this the FOIL method…(First, Outer, Inner, Last)

Ex 3:

$(x + 4)(x + 2)$

Ex 4:

$(2x + 4)(x – 3) =$

***Note: FOIL only works with 2 binomials…what if we have BIGGER polynomials?**

Ex 5:

$(x + 3)(-x^2 + 3x + 2)$

Ex 6:

$(x^3 + 4x – 1)(x – 2)$

10.3 "Special Products of Polynomials"

***FOIL always works to multiply binomials, but some pairs of binomials have special products, and you can take a shortcut by memorizing the pattern!**

Sum and Difference Pattern: $(a + b)(a – b) = a^2 – b^2$
*There is no middle term!

Ex1: $(x + 4)(x – 4) =$ Ex2: $(3x – 5)(3x + 5) =$ Ex3: $(x + y)(x – y) =$

Square of an Addition Binomial: $(a + b)^2 = a^2 + 2ab + b^2$
*The middle term is twice the product of the two terms!

Ex4: $(x + 5)^2 =$ Ex5: $(2x + 4)^2 =$ Ex6: $(x + y)^2 =$

Square of a Subtraction Binomial: $(a – b)^2 = a^2 – 2ab + b^2$
*The middle term is Negative twice the product of the two terms!

Ex7: $(x – 2)^2 =$ Ex8: $(2x – 5)^2 =$ Ex9: $(x – y)^2 =$

10.4 "Solving Polynomial Equations in Factored Form"

Standard Form: $2x^2 + 7x – 15 = 0$
→ We can solve this using the Quadratic Formula!

Factored Form: a polynomial is in factored form if it is written as the product of two or more linear factors

Factored Form: $(2x – 3)(x + 5) = 0$
→ How do we solve this?
*We use the Zero Product Property!

Zero Product Property- the product of two factors is zero only when at least one of the factors is zero!
If ab = 0, then either a = 0 or b = 0

Ex 1: If $(2x – 3)(x + 5) = 0$, then either $2x – 3 = 0$ or $x + 5 = 0$

Ex 2: $y = 5x(x - 3)(2x + 6)$

Ex 3: $y = 3(2x - 5)$

***If you have "repeated factors'" you only have to solve the factor once….**

Ex 4: $(x + 3)^2 = 0$

Ex 5: $(2x - 6)^3 = 0$

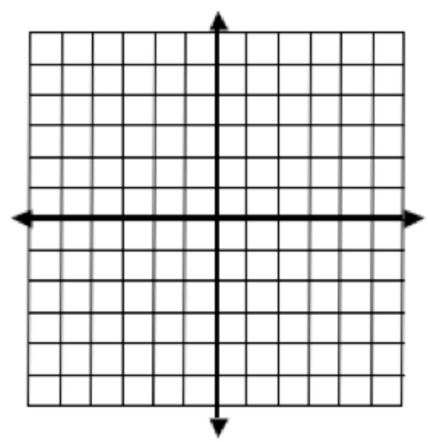
***You will have as many solutions for x as the number of factors you have in your factored equation!**

Ex 6: $(2x + 3)(x + 4)(3x - 5) = 0$

Ex 7: $8(2x + 1)(x - 5)^2 = 0$

***When you factor an equation the solutions are the roots to the graph!**

Ex 8: Solve $(x - 3)(x + 2) = 0$



Write $(x - 3)(x + 2) = 0$ in standard form:

The parabola opens ______

AOS: _________

Vertex: __________

10.8 "Factoring GCF & Grouping"

<u>GCF (greatest common factor)</u>

Integer and/or variable factors that can be factored out of **all** terms in a polynomial expression.

1) Take out the GCF of the coefficients
2) Take out the minimum variables available in each term
3) Reverse the distributive property, the "leftovers" go in parentheses

Ex 1: Factor $15x^4 + 3x^2$

GCF of 15 & 3: ____

Minimum x-values in each term: ____

The GCF of the polynomial = ____

Ex 2: Factor $6x^3 + 18x^2 - 24x$

Ex 3: Factor $3x^2 - 5$

Prime Polynomial- a polynomial that is NOT the product of multiple polynomials having integer coefficients (it can't be nicely factored)

Factoring by Grouping- when there are four terms, try factoring by grouping!

1) Factor GCF
2) Factor by grouping the first two terms and the last two terms

Ex4: Factor $x^3 + 2x^2 + 3x + 6$

There is no GCF that can be pulled out of all 4 terms, but the first 2 terms have a common factor, and the second 2 terms have a common factor.

$$x^3 + 2x^2 + 3x + 6$$

***Notice that the terms in parentheses are THE SAME! This should always be the case when you factor by grouping, and if they are NOT the same, something is WRONG!**

*The terms need to be the same, because you can now re-write the answer as two factors multiplied together, using the Distributive Property:

Ex5: Factor $9x^2 - 15x + 6x - 10$ (Remember to check for GCF first!)

Ex 6:
Factor $4x^4 + 10x^3 - 12x - 30$ (Check for a GCF first!)

Factor the following:

Ex 7:
$18x^2 - 15x$

Ex 8:
$x^4 + 2x^3 + x + 2$

Ex 9:
$x^4 + 5x^3 + 3x - 15$

Ex 10:
$10x^7 - 5x^5 + 15x^3 - 20x^2$

10.5 "Factoring $x^2 + bx + c$"

<u>Factor a Quadratic Expression</u>- Write it as the product of 2 linear expressions

*FOIL tells us that $(x + 2)(x + 3) = x^2 + 5x + 6$
We are trying to factor a polynomial: $x^2 + bx + c$ so that $b = 5 = 2 + 3$ and $c = 6 = (2)(3)$

*We are looking for 2 numbers that when added together give us the x-term (b), and when multiplied together give us the constant term (c) .

<u>HINT</u>: Look at the last term and list the factors…
- If the last term is POSITIVE, both numbers must be + or both must be - because (+)(+) = + and (-)(-) = +
- If the last term is NEGATIVE, one number must be + and the other must be –

<u>Ex 1</u>: Factor $x^2 + 7x + 10$

*Which 2 number when multiplied give us +10, and when added give us +7?

You can check by FOILing the factors = $x^2 + 7x + 10$

<u>Ex 2</u>: Factor $x^2 – 7x + 12$

<u>Ex 3</u>: Factor $x^2 – 6 + x$

<u>Ex 4</u>: Factor $– 10 + x^2 – 3x$

<u>Ex 5:</u> Factor $x^2 + 3x – 6$

*<u>**NOTE**</u>- A Quadratic Polynomial is sometimes PRIME (it cannot be factored).

Solve by factoring.
Remember to write in standard form 1st! $ax^2 + bx + c = 0$

<u>Ex 6:</u> $x^2 – 8x + 15 = 0$

<u>Ex 7:</u> $x^2 – 2x = 24$

10.6 "Factoring $ax^2 + bx + c$"

*Last time we learned to factor $x^2 + bx + c$, where the coefficient on the x^2 term was ALWAYS 1…now it may not be 1 anymore!

<u>The "Super Cool Method" for Factoring Quadratic Polynomial when $a \neq 1$:</u>

1) Write the polynomial in standard form, with all terms on the left side (set it equal to 0)
2) Simplify by pulling out any GCF from all terms (if any exist)
3) Multiply (a)(c)
4) List the factors of (a)(c) that when added would equal b term
5) Rewrite the polynomial with the two terms for "b" in the center (with "x" variable)
6) Factor the polynomial by GROUPING

<u>Ex 1</u>: Factor $2x^2 + 11x + 5$

<u>Ex 2</u>: Factor $3x^2 – 4x – 7$

<u>Ex 3</u>: Factor $6x^2 – 19x + 15$

<u>Ex 4</u>: Factor $6x^2 – 2x – 8$ ***We can simplify by pulling out a GCF!**

*AGAIN, if you are struggling to factor a quadratic polynomial, it might be PRIME!

10.7 "Factoring Special Products"

***We can use our special product patterns to factor polynomials!**

1) Difference of 2 Squares Pattern: $a^2 - b^2 = (a + b)(a - b)$

Ex 1: $4x^2 - 25 =$

2) Perfect Square Trinomial (Addition): $a^2 + 2ab + b^2 = (a + b)^2$

Ex 2: $x^2 + 6x + 9 =$

3) Perfect Square Trinomial (Subtraction): $a^2 - 2ab + b^2 = (a - b)^2$

Ex 3: $x^2 - 10x + 25 =$

***Remember to always try GCF first!**

Ex 4: Factor and Solve $50 - 98x^2$

Ex 5: Factor and Solve $x^2 - 4x + 4$

Ex 6: Factor and Solve $16x^2 + 24x + 9$