Algebra 1: *Hutschenreuter* **Name** _____
Chapter 11 Note Packet

11.1 "Ratio and Proportion"

Proportion: an equation that states that two ratios are equal

$\frac{a}{b} = \frac{c}{d}$ $b \neq 0, d \neq 0$ "a is to b as c is to d"

Extremes: a and d (Top left, bottom right)
Means: b and c (Bottom left, top right)

Properties of Proportions:

- Reciprocal Property: If 2 ratios are equal, their reciprocals (if they exist) are also equal

  If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$ $\quad$ *If* $\frac{1}{2} = \frac{3}{6}$, *then* $\frac{2}{1} = \frac{6}{3}$

- Cross Product Property: The product of the extremes equals the product of the means

  If $\frac{a}{b} = \frac{c}{d}$, then ad = bc $\quad$ *If* $\frac{1}{2} = \frac{3}{6}$ then (__)(__) = (__)(__)

*We can use these properties to solve proportions that have a single variable!

Ex 1: $\frac{x}{5} = \frac{9}{11}$ $\quad$ Ex 2: $\frac{p}{8} = \frac{2}{p}$

*What if one of the solutions doesn't work? THIS CAN HAPPEN!
Extraneous Solution: a solution that does not satisfy the original equation

Ex 3: $\frac{2x-3}{4x} = \frac{x-1}{x}$

Ex 4:
I have a painting that is 2ft by 5ft, but would like to have an enlargement made that is 5 feet wide. What will the length be?

Ex 5:
To estimate the deer population at Green Park, rangers caught and tagged 52 deer and released them. Later they captured 85 deer and 32 out of 85 were tagged. Approximately how many deer live in Green Park?

11.2 "Percents"

Two Ways to Solve Percent Problems:

1) Set up an equation:

Number being compared to base number	is	%	of	Base Number
(a)	=	(P)	•	(b)

*The algebraic model is always **a = (p)(b)**, where b is the base number

2) Set up a proportion:

$\frac{\%}{100} = \frac{is}{of}$ Solve by cross-multiplying

Ex 1)
What is 5% of 32?

Ex 2)
Seven is 13% of what?

Ex 3)
Three is what % of 19?

Ex 4)
From a school survey 78% of students said they eat hot lunch. If 945 students attend Logan High School how many eat hot lunch?

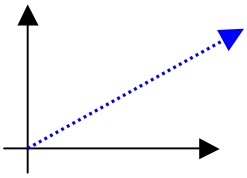
Ex 5)
Forty-five percent of teachers and staff at Central and Logan High said they were interested in merging the two schools into one high school. This means 135 teachers and staff are interested. How many teachers and staff combined work at the two high schools?

11.3 "Direct and Inverse Variation"

*Recall that "k" is the constant of variation
(K is like the slope of a slope-intercept equation y = mx + b where the y-intercept (b) is ALWAYS zero)

Direct Variation

What it looks like:



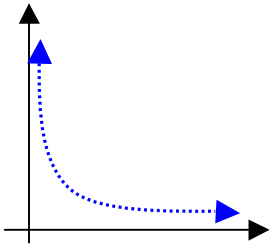
-Straight line
-Contains (__, __)
-How to find K: $k = \frac{y}{x}$
-How to write Direct Variation Equation: y = kx
-In Science _____ Law is direct variation

Ex 1: x and y vary directly.
Let x = 4 & y = 8. Find k & write an equation.

Ex 2: x and y vary directly.
Let x = 45 & y = 81. Find k & write an equation.

Inverse Variation

What it looks like:



-Decreasing curved line
-Looks like exponential decay
-How to find K: k = xy = k
-How to write Inverse Variation Equation: $y = \frac{k}{x}$
-In Science _____ Law is inverse variation

Ex 3: x and y vary inversely.
Let x = 4 & y = 8. Find k & write an equation.

Ex 4: x and y vary inversely.
Let x = 12 & y = ¾. Find k and write an equation.

11.4 "Simplifying Rational Expressions"

Rational Number- a number that can be written as the quotient of two integers

*A fraction whose numerator, denominator, (or both) are non-zero polynomials is a Rational Expression

Undefined Expression- When the denominator is equal to zero.

Ex 1: $\frac{2}{x+1}$ $x \neq -1$, but can be any other real number!

*To simplify a fraction, simplify the numerator and the denominator, then divide out any common factors (those appearing on the top and the bottom).

Examples: Simplify the expressions

2) $\frac{3x}{6x+9} =$

3) $\frac{x^3+4x}{x} =$

4) $\frac{3x+1}{x} =$

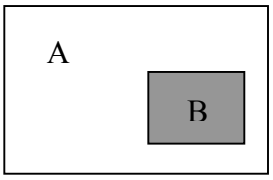
Recognizing "Opposite Factors":

Ex 5: $\frac{4-x^2}{x^2-x-2} =$

*Opposite Factors are (2-x) and (x-2), So you can "pull out" a –1: -1(-2 + x) = -1(x – 2)

Geometric Probability- Region B is contained in Region A. An object is tossed and is equally likely to land at any point in the region. The probability that it lands in Region B is:

$P = \frac{\text{Area of B}}{\text{Area of A}}$



11.5 "Multiplying and Dividing Rational Expressions"

*Finding Products and Quotients of Rational Expressions is JUST LIKE finding products and quotients of fractions!

→ To MULTIPLY Rational Expressions, multiply straight across the top and straight across the bottom, then simplify

→ To DIVIDE Rational Expressions, flip the 2nd expression over and follow the rules of multiplying! (Multiply by the reciprocal of the divisor)

Ex 1: Multiplying MONOMIAL expressions

$$\frac{9x^2}{4} \cdot \frac{8}{18x} =$$

Ex 2: Multiplying POLYNOMIAL Expressions

$$\frac{3x}{x^2 - 2x - 24} \cdot \frac{x - 6}{6x^2 + 9x} =$$

Ex 3: Dividing POLYNOMIAL Expressions

$$\frac{x^2 - 8x + 15}{x^2 - 3x} \div \frac{3x - 15}{1} =$$

11.6 "Adding and Subtracting Rational Expressions"

*To ADD or SUBTRACT expressions with the **SAME** DENOMINATORS:

- Combine the numerators
- Write the result over the common denominator
- SIMPLIFY!

Adding $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$ Subtracting $\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}$

Ex 1: $\frac{3}{x+1}+\frac{2x+2}{x+1}=$

Ex 2: $\frac{4x}{3x^2-x-2}-\frac{(x-2)}{3x^2-x-2}=$

*To ADD or SUBTRACT expressions with **DIFFERENT** Denominators:

- Find a COMMON DENOMINATOR (Usually the least common multiply of the 2 original denominators)
- Change the NUMERAOTS to make Equivalent Expressions
- Add/subtract the Numerators
- SIMPLIFY!

Adding $\frac{a}{b}+\frac{c}{d}=\frac{a(d)+c(b)}{(b)(d)}$ Subtracting $\frac{a}{b}-\frac{c}{d}=\frac{a(d)-c(b)}{(b)(d)}$

Ex 3: $\frac{2x^2}{5x}+\frac{2}{2x^2}=$

Ex 4: $\frac{(x+4)}{x^2-1}+\frac{2x}{x+1}=$

Ex 5: $\frac{4x}{3x-2}-\frac{2x}{3x+1}=$

11.7 "Dividing Polynomials"

*To divide a polynomial by a monomial, divide each term by the monomial, then simplify each fraction:

Ex 1: $(4x^3 + 2x^2 - x + 5) \div 2x =$

*To divide a polynomial by a binomial, we have to use long division…remember long division?

Try $465 \div 38$

Dividend- the number under the division bar
Divisor- the number outside the division bar

*When dividing polynomials, both the dividend and divisor must be in standard form (descending order). If a term is not present, "hold the place" with a zero!

Ex 2: $x + 2\overline{)5x^2 + 2x + 3}$

*Use your factoring skills: what # times x gives me $5x^2$?
*Always put the first expression above the division bar over the 2nd term, because we are dividing by a binomial!

Ex 3: $x - 3\overline{)3x^4 + 2x^3 + 0x^2 + 5x + 1}$

11.8 "Rational Equations and Functions"

*When solving SIMPLE Rational Equations (One term on each side): Cross Multiply!

Ex 1: $\frac{5}{x+4} = \frac{5}{3(x+1)} \rightarrow$

*When solving more complicated Rational Equations (More than one term on each side): Multiply all terms on both sides by the LCD!

Ex 2: $\frac{x}{x+3} = \frac{3}{x+3} + 2$ LCD = _____

Ex 3: $\frac{2}{x} + \frac{1}{3} = \frac{4}{x}$ LCD = _____

*Remember you can check your answer by plugging it into the equation and checking to make sure both sides are equal!
*Note: Sometimes you may need to FACTOR the denominators to find the LCD!

*GRAPHING rational functions of the form : $y = \frac{a}{x-h} + k$
These functions are called HYPERBOLAS, with center at (h,k).
Asymptote- a line that the graph approaches but never crosses

Graphing Rational Functions:
1) Make sure the function is in the correct form (You may have to "long divide" the polynomials to get it in the correct form)
   Ex: $y = \frac{2x+1}{x+2}$ ….divide 2x + 1 by x + 2…. $x+2\overline{)2x+1}$ $\frac{2}{} + \frac{-3}{x+2} = \boxed{2 + \frac{-3}{x+2} \text{ OR } \frac{-3}{x+2} + 2}$
2) Find and label the center (h, k) and asymptotes (the lines x = h and y = k)
3) Make a table of (x,y) values, including x-values less than and greater than h.
4) Plot the points and connect the lines with smooth curves

Ex: Graph $y = \frac{-3}{x+2} + 2$

Center = _____

Asym: x = _____, y = _____