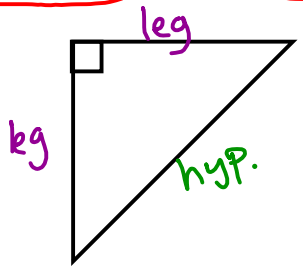
# Section 4.4: Prove Triangles Congruent by SAS and HL

EQ: How can you use 2 sides and an ∠ to prove △s are ≅?

Consider a relationship involving two sides and the angle they form, their *included* angle. To picture the relationship, form an angle using two pencils.


**Vocabulary:**

**leg of a right triangle**
The two legs create the right angle




**hypotenuse**
(hyp.) opposite of the right angle
* largest side of a rt. △

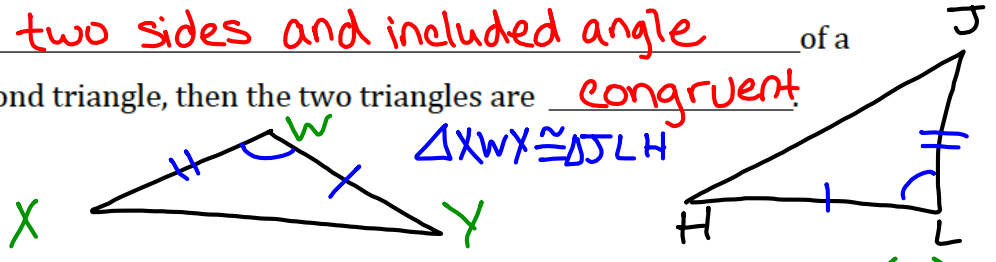
**included angle**
The angle created by two given sides

1) Draw triangle with sides 2 inches, 8.5 cm, and the included angle of 65 degrees.
2) Cut out triangle ABC...compare with others.

# Does a Side-Angle-Side Congruence pattern guarantee two congruent triangles?

**Postulate 20: Side-Angle-Side Congruence Postulate** (SAS)

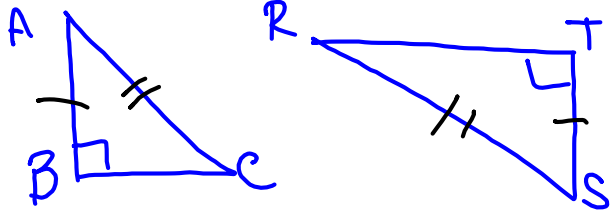
If two sides and the included angle of one triangle are congruent to two sides and included angle of a second triangle, then the two triangles are congruent.




$\triangle XWY \cong \triangle JLH$

**Theorem 4.5: Hypotenuse-Leg Congruence Theorem** (HL)

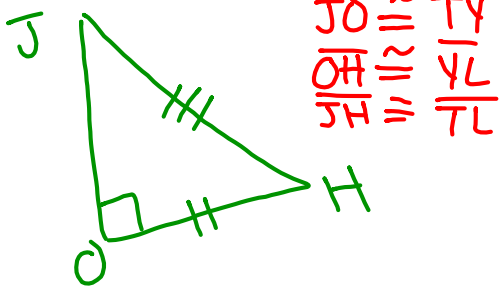
If the hypotenuse and a leg of one right triangle are congruent to the hyp. and leg of a second triangle, then the two triangles are congruent.



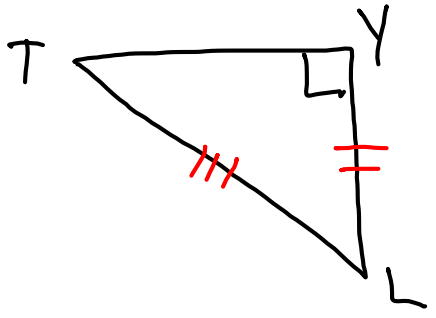

$\triangle ABC \cong \triangle STR$

A1. Sketch $\Delta JQH$ and $\Delta TYL$ that are congruent by HL.

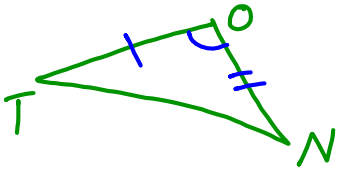
only 1 w/ rt. △



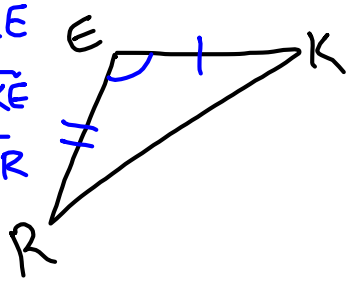

$\overline{JQ} \cong \overline{TY}$
$\overline{QH} \cong \overline{YL}$
$\overline{JH} \cong \overline{TL}$



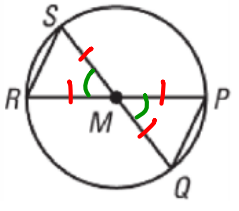

A2. Sketch $\Delta TQN$ and $\Delta KER$ that are congruent by SAS.




$\angle Q \cong \angle E$
$\overline{TQ} \cong \overline{KE}$
$\overline{QN} \cong \overline{ER}$




A3. Why is $\Delta SMR \cong \Delta PMQ$? ($M$ is the center of the circle)

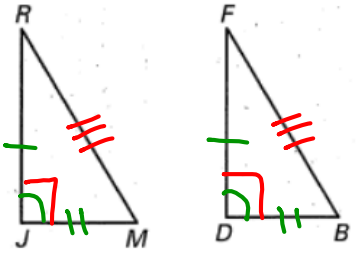



$\overline{MS}$
$\overline{MR}$
$\overline{MP}$
$\overline{MQ}$
} all radius ∴ all ≅

$\angle SMR \cong \angle QMP \rightarrow VA$

$\Delta SMR \cong \Delta QMP \rightarrow SAS$

**A4. State the third congruence that must be given to prove $\Delta JRM \cong \Delta DFB$.**




a. $\overline{JR} \cong \overline{DF}$, $\overline{JM} \cong \overline{DB}$, $\angle J \cong \angle D$ (by SAS)

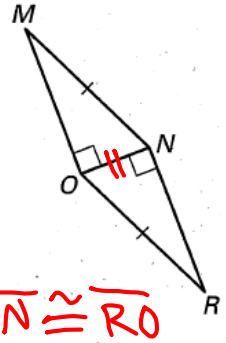
*Green marks

b. $\overline{MR} \cong \overline{BF}$, $\angle J$ is a right angle, $\angle J \cong \angle D$, ← ∴ rt ∠s $\overline{JM} \cong \overline{DB}$ or $\overline{RJ} \cong \overline{FD}$ → not both (by HL)

*Red Marks

A5. Decide if there is enough information to prove that the two triangles are congruent. If there is, give a congruence statement for the triangles and the reason(s) for the congruence.

a.

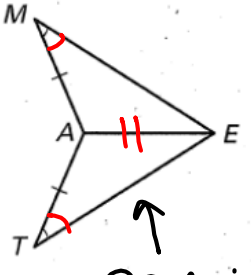



$\overline{MN} \cong \overline{RO}$ (hyp.)

$\overline{NO} \cong \overline{ON}$ Reflexive

$\Delta MON \cong \Delta RNO$ HL

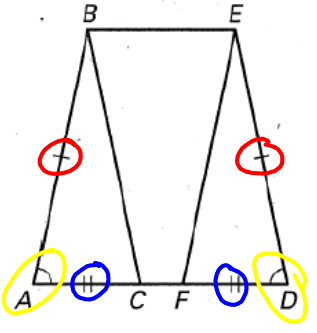
b.




SSA is not a $\cong$ thm.

∴ $\Delta$s $\ncong$

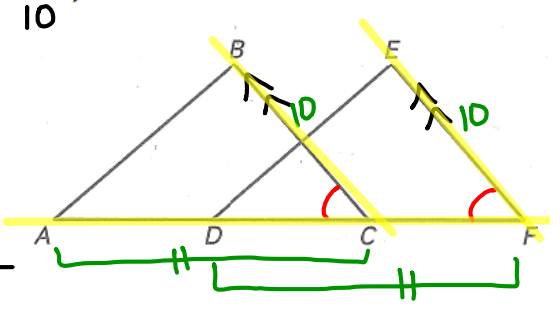
c.




$\Delta BAC \cong \Delta EDF$

SAS $\Delta\cong$

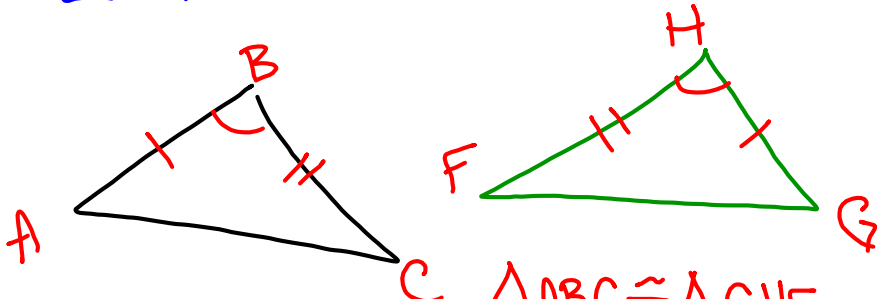
**A6. Given:** $\overline{CB} \parallel \overline{FE}, \overline{AC} \cong \overline{DF}, BC = 2\cancel{5}\ 10$, **and** $EF = 10$
**Prove:** $\Delta ABC \cong \Delta DEF$




| Statement | Reason |
|---|---|
| $\overline{CB} \parallel \overline{FE}$ | Given |
| $\overline{AC} \cong \overline{DF}$ | Given |
| $BC=10, EF=10$ | Given |
| $BC=EF$ | subst. |
| $\overline{BC} \cong \overline{EF}$ | Def. of $\cong$ |
| $\angle BCA \cong \angle EAD$ | CA $\cong$ thm. |
| $\Delta BCA \cong \Delta EAD$ | SAS $\Delta \cong$ |
| | |

# 4.4 Summary:

If 2 sides and the included $\angle$ of one $\triangle$ are $\cong$ to another $\triangle$ then the $\triangle$'s are $\cong$.




$\triangle ABC \cong \triangle GHF$
SAS