Section 9.1: Translate Figures & Use Vectors

EQ: How do you translate (shift) a figure using a vector?

KEY VOCABULARY

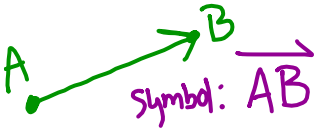
image — End with
A new figure by transformation(s)

preimage — Start with
The original figure

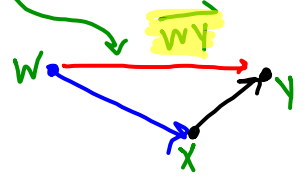
isometry — * stays same shape and size
A congruence transformation

translation — shift
A move of every pt. of a figure in the same direction & same distance

vector
A quantity that has direction and size (magnitude)

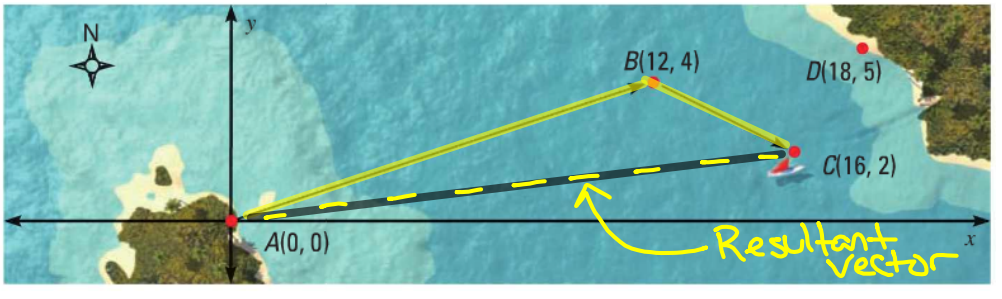


resultant vector
A vector sum of two or more vectors



9.1 Translate Figures and Use Vectors

NAVIGATION A boat heads out from point A on one island toward point D on another. The boat encounters a storm at B, 12 miles east and 4 miles north of its starting point. The storm pushes the boat off course to point C, as shown.



Vector coordinate form

(x, y) --> (x + h, y + k)

(−) left
(+) right
(−) Down
(+) UP

Vector component form

$\langle h, k \rangle$

Where h is the horizontal component and k is the vertical component

Theorem 9.1: Translation Theorem

A translation is an isometry.

A1. Use the translation $(x, y) \rightarrow (x + 2,\ y - 7)$. Right 2 units, down 7 units

a. What is the image of $A(6, -3)$? (preimage) +2 −7 $A'(8, -10)$ ← read "A prime"

b. What is the image of $B(0, 9)$? +2 −7 $B'(2, 2)$

c. What is the preimage of $C'(-10, 42)$? "undo" −2 +7 $C(-12, 49)$

d. What is the preimage of $D'(4f, -g)$? −2 +7 $D(4f-2, -g+7)$ * an expression

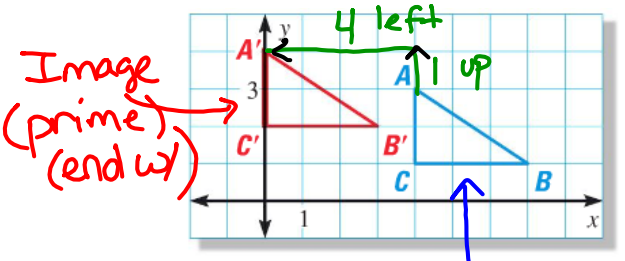
A2. Write a coordinate rule $(x+h, y+k)$ for the translation.

a. 4 units to the left and 3 units up. → x−, y+ $(x,y) \rightarrow (x-4, y+3)$

b. 11 units down. → y− * NO x+h because not moving lt./rt. $(x,y) \rightarrow (x, y-11)$

EXAMPLE 2

Write a rule for the translation of $\triangle ABC$ to $\triangle A'B'C'$. Then verify that the transformation is an isometry.



coordinate form... $(x,y) \rightarrow (x-4, y+1)$

preimage (start)

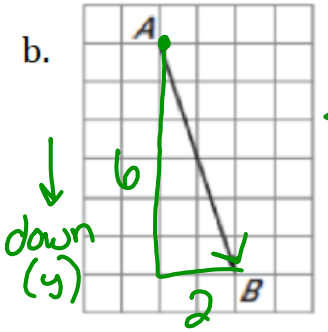
component form.... $\langle -4, 1 \rangle$

A3. Name the vector and give the vector's component form.

a.

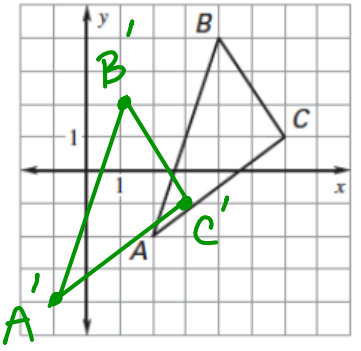
$\overrightarrow{VW}$ $\langle -7, -3 \rangle$

b.



$\overrightarrow{AB}$ $\langle 2, -6 \rangle$

A4. Graph the transformation of preimage ΔABC over $\langle -3, -2 \rangle$.



3 left, 2 down

$A(2,-2) \rightarrow A'(-1,-4)$ (−3, −2)

$B(4,1) \rightarrow B'(1,2)$ (−3, −2)

$C(6,1) \rightarrow C'(3,-1)$ (−3, −2)

A5. A cross country skier travels from A to D. At point B and C, the skier changes directions as shown. The distances are in miles.

a. Write the component form of $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

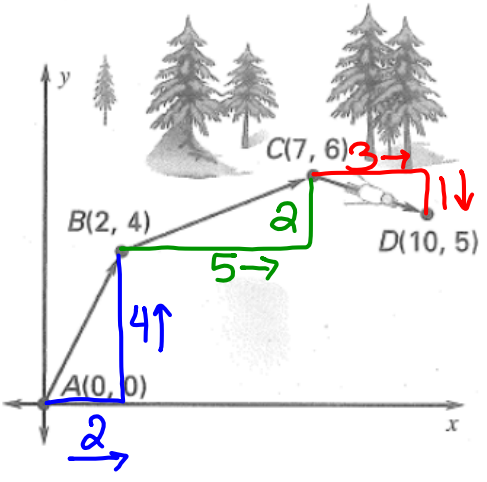
$\langle 7-2, 6-4 \rangle$

$\langle 5, 2 \rangle$

$\langle 2, 4 \rangle$

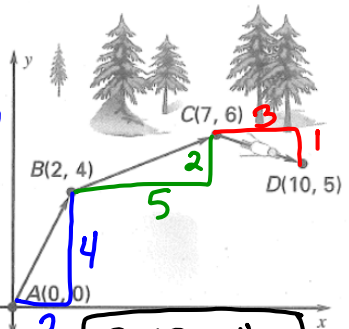
b. Write the coordinate form of $\overrightarrow{CD}$.

$(x,y) \rightarrow (x+3, y-1)$



A5. A cross country skier travels from A to D. At point B and C, the skier changes directions as shown. The distances are in miles.

c. What is the total distance the skier traveled?



$AB = \sqrt{2^2+4^2} = \sqrt{4+16} = \sqrt{20} \approx 4.47$

$BC = \sqrt{5^2+2^2} = \sqrt{25+4} = \sqrt{29} \approx 5.39$

$CD = \sqrt{3^2+1^2} = \sqrt{9+1} = \sqrt{10} \approx 3.16$

Add: $+$ 13.02 miles

d. If the skier went from A to D, what would be the distance?

A(0,0) to D(10,5)

$x \rightarrow 10-0=10$

$y \rightarrow 5-0=5$

$\therefore AD = \sqrt{10^2+5^2}$

$\sqrt{100+25} = \sqrt{125}$

$\approx$ 11.18 mi

9.1 Summary:

A figure is translated (shift) using a vector (direction), which describes the horizontal movement (h) and the vertical movement (k)

(x+h, y+k)

if +h move right, if -h move left

if +k move up, if -k move down