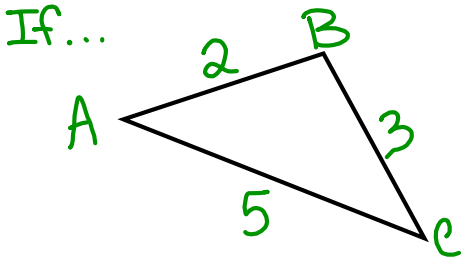
Section 5.5: Use Inequalities in a Triangle

EQ: How do you find the possible lengths of the 3rd side of a △ given the other 2 sides?

Theorem 5.10: If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.

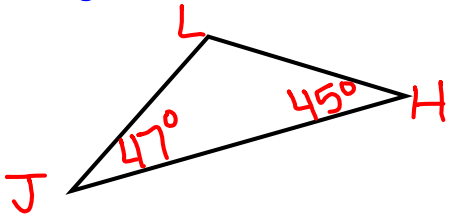


then... $2 < 3 < 5$

thus $m\angle C < m\angle A < m\angle B$

smallest ∠ largest ∠

Theorem 5.11: If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.



$47° > 45°$

$\therefore LH > JL$

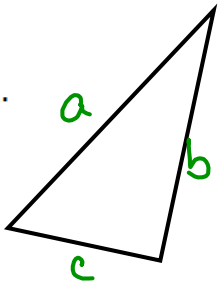
Theorem 5.12: Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the 3rd side.

$a + b > c$

$b + c > a$

$a + c > b$

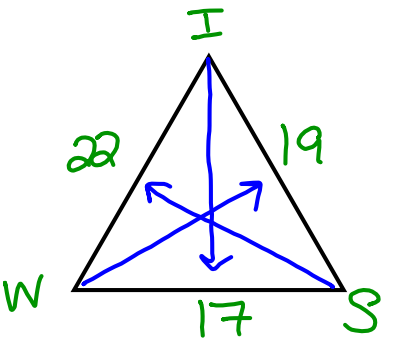


A1. ΔWIS has side lengths of $WI = 22, WS = 17,$ and $SI = 19$. List the angles from smallest measure to largest.

∠S ∠I ∠W

$17 < 19 < 22$

$\angle I, \angle W, \angle S$



*Trick/shortcut: $\overline{WS}$ corresponds to $\angle I$ because I is not the endpt.

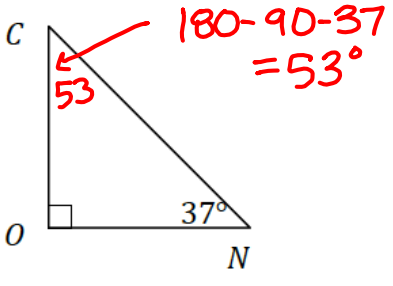
$\overline{WI} \rightarrow \angle S$ *The vertex not listed is the $\angle$
$\overline{SI} \rightarrow \angle W$

A2. In ΔCON list the sides in order from longest to shortest.

$90° > 53° > 37°$

$\angle O, \angle C, \angle N$

$\overline{CN}, \overline{ON}, \overline{CO}$



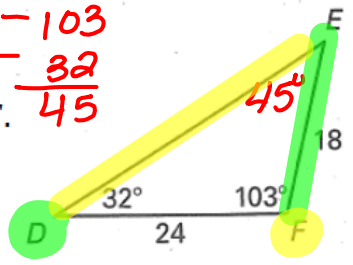
A3. Can you make a triangle with side lengths of 11 inches, 17 inches, and 5 inches? Explain.

NO, $5+11=16$ $16<17$

Thm 5.12: Δ inequality says sum has to be > 3rd side

A4. Use the following diagram.

180
−103
− 32
45



a. Name the smallest and largest angles of ΔDEF.

↳ ∠D ↳ ∠F

b. Name the shortest and longest sides of ΔDEF.

∠D corresponds to ↓ $\overline{EF}$ (shortest)

∠F corresponds to ↓ $\overline{DE}$ (longest)

A5. Describe the possible length of the third side of a triangle given the lengths of the other two sides.

a. 12 cm, 10 cm

12+10=22

12−10=2

Between 2cm to 22cm

b. 29.5 ft , 55 ft

55+29.5=84.5

55−29.5=25.5

$25.5' < \ell < 84.5'$

c. 2 ft , 32 in

↓

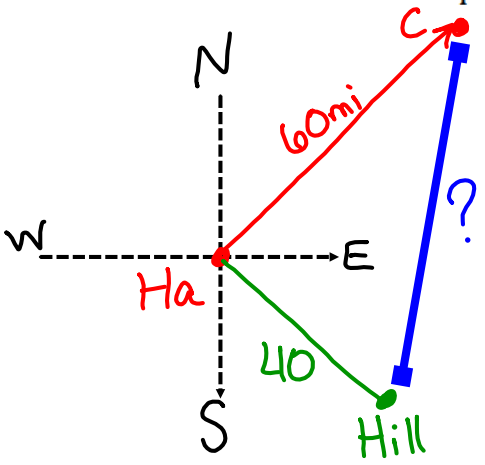
24 in

32+24=56

32−24=8

Between 8" and 56"

A6. Crayton Falls is 60 miles NE of Harnedville. Hill City is 40 miles SE of Harnedville. How far apart are Crayton Falls and Hill City?

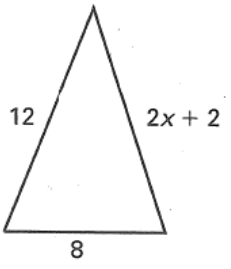


$60+40=100$

$60-40=20$

20 mi to 100 mi apart

A7. What are the possible values for x?



$12+8=20$

$12-8=4$

$4 < 2x+2 < 20$

$-2 \quad -2 \quad -2$

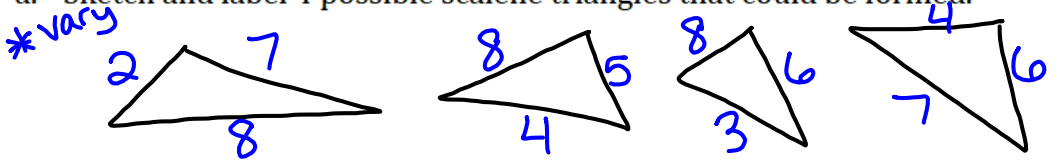
$\frac{2}{2} < \frac{2x}{2} < \frac{18}{2}$

$1 < x < 9$

*The value of x is between 1 and 9

A8. You have a 17-inch piece of wire. You need to bend the wire to form a triangle with whole number side lengths.

a. Sketch and label 4 possible scalene triangles that could be formed.



b. List two combinations of side lengths that will not produce triangles.

* sum has to be > 3rd side

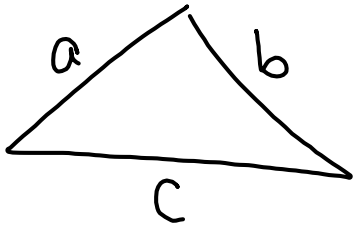
1, 1, 15 → 1+1 = 2 < 15

2, 2, 13 → 2+2 = 4 < 13

2, 3, 12 → 2+3 = 5 < 12

5.5 Summary:

Given 2 side lengths of a triangle the 3rd side is greater than the difference (minus) and is less than the sum (add) of the 2 given sides lengths.



$c < a + b$ and $c > |a - b|$

OR....

$|a - b| < c < |a + b|$