Section 9.3
Law of Sines

Area of a triangle

$$K = \tfrac{1}{2}\,ab\sin C = \tfrac{1}{2}\,bc\sin A = \tfrac{1}{2}\,ac\sin B$$

$\tfrac{1}{2}abc$ $\tfrac{1}{2}abc$ $\tfrac{1}{2}abc$

*Divide everything by $(\tfrac{1}{2}abc)$

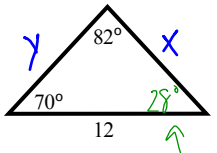
Law of Sines $\dfrac{\sin C}{c} = \dfrac{\sin A}{a} = \dfrac{\sin B}{b}$

When do you use Law of Sines?

ASA / AAS / SSA

Example 1

Find all missing sides and angles.



$180 - 82 - 70 = 28°$

$\dfrac{\sin 82°}{12} = \dfrac{\sin 70}{x}$

$x = \dfrac{12(\sin 70)}{\sin 82}$

$x = 11.39$ un

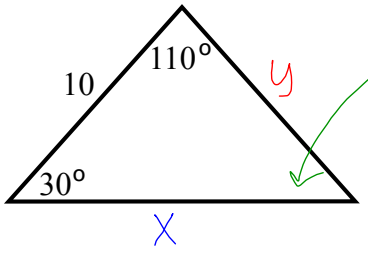
$\dfrac{\sin 82}{12} = \dfrac{\sin 28}{y}$

$y = \dfrac{12(\sin 28)}{\sin 82}$

$y = 5.69$ un

Example 2

Find all missing sides and angles.



$180 - 110 - 30 = 40°$

$m\angle = 40°$

$\dfrac{\sin 40°}{10} = \dfrac{\sin 110°}{x}$

$x = \dfrac{10(\sin 110°)}{\sin 40°}$

$x = 14.62$ un

$\dfrac{\sin 40°}{10} = \dfrac{\sin 30°}{y}$

$y = \dfrac{10(\sin 30°)}{\sin 40°}$

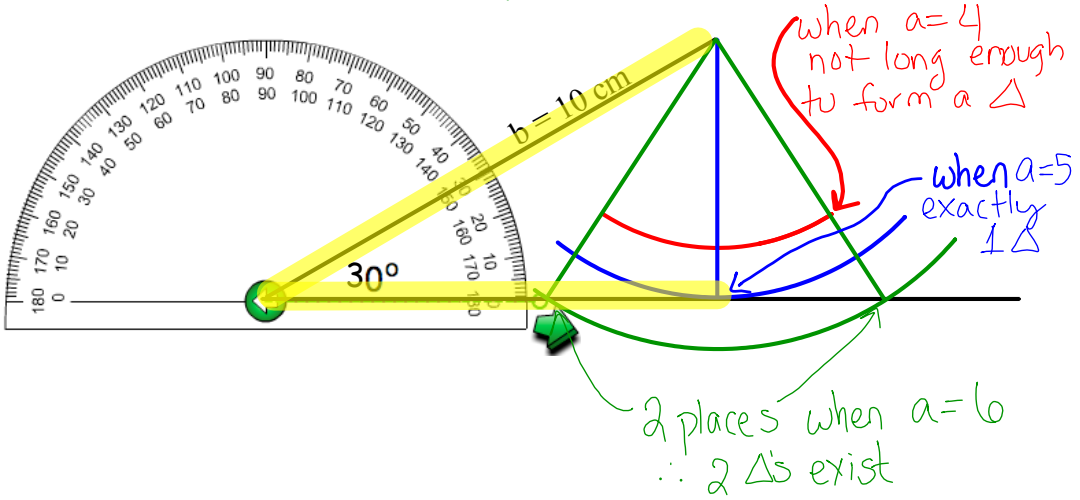
$y = 7.78$ un

Construct the following three triangles.

1) A = 30° b = 10 a = 4 NO △ exists

2) A = 30° b = 10 a = 5 1 △ exists

3) A = 30° b = 10 a = 6 2 △'s exist



Test for number of possible triangles * only need if given *SSA

Note: the value on the left side of the inequality is opposite of the angle

side opposite of given angle (side/side/Angle)

1) a < bsinA then **no** triangle exist

2) a = bsinA then **one** triangle exist

3) a > bsinA and a < b then **two** triangles exist

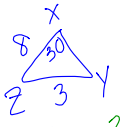
4) a > bsinA and a ≥ b then **one** triangle exist

Example 3

ΔXYZ X = 30°

x = 3 ← side opp. of ∠X so goes on left side

y = 8



*SSA 3 [?] 8·sin30°

3 < 4 ∴ No △ exists

Try to solve...

$\frac{\sin 30^\circ}{3} = \frac{\sin Y}{8}$

$\sin Y = \frac{8(\sin 30^\circ)}{3}$

$\sin Y = 12$

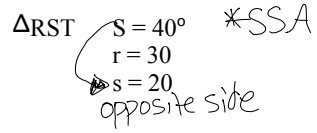
$m\angle Y = \sin^{-1}(12) = \text{D.N.E.}$

"Domain error"

so you know there's NO △ possible

Example 4

How many triangles exist? If any triangle(s) exist find all sides and angles.



*SSA

$20 \; ? \; 30 \cdot \sin 40°$

$20 > 19.28$ AND $20 < 30$

so 2 Δ's exist!

① $\frac{\sin 40°}{20} = \frac{\sin R}{30}$

$\sin R = \frac{30(\sin 40°)}{20} = .964$

② $\sin^{-1}(.964) = m\angle R = 74.6°$

③ $m\angle T = 180 - R - S = 180 - 40 - 74.6$

$m\angle T = 65.4°$

④ $\frac{\sin 65.4°}{t} = \frac{\sin 40°}{20}$

⑤ $t = \frac{20(\sin 65.4)}{\sin 40°}$

$t = 28.29$ un

⑥ How to get 2nd Δ...

$180 - 74.6° = 105.4°$

⑦ so Δ2 → $m\angle R = 105.4°$

then $180 - R - S = m\angle T$

⑧ $m\angle T = 180 - 105.4 - 40 = 34.6°$

⑨ $\frac{\sin 34.6°}{t} = \frac{\sin 40°}{20}$

⑩ $t = \frac{20(\sin 34.6)}{\sin 40°}$

$t = 17.67$ un