

ESSENTIAL QUESTION: How can you find the area of a regular pentagon?

QUESTIONS:

Vocabulary of Regular Polygons:

Center

Radius from center to vertex

Apothem

(a)  $\perp$  from center to side

Central Angle ( $\theta$ )

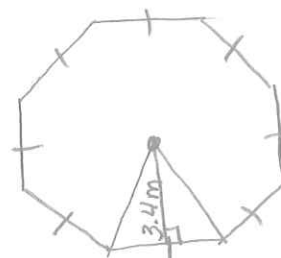
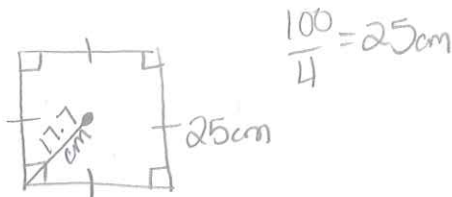
Vertex of angle is the center

A1. Sketch the following.

a. A regular quadrilateral with a perimeter of 100 cm and a radius of about 17.7 cm.

b. A regular octagon with an apothem of about 3.4 m.

equiangular and equilateral  $\therefore$  SQUARE



Theorem 11.11: Area of a Regular Polygon

The area of a regular  $n$ -gon with side length  $s$  and apothem  $a$  is one-half the product of  $s$ ,  $a$ , and  $n$ . (Also...the area of a regular  $n$ -gon with perimeter  $P$  is one-half the product of  $P$ , and  $a$ .)

$$\text{Area} = A = \frac{1}{2} s \cdot a \cdot n \quad \text{or} \quad A = \frac{1}{2} P \cdot a$$

A2. Name or find the measure of the part of the regular polygon.

a. Center

b. Radius

c. Apothem

G

or  $\frac{GB}{GA}$

$\frac{GH}{GH}$

d.  $m\angle AGB$

e.  $m\angle HGA$

f.  $m\angle BHG$

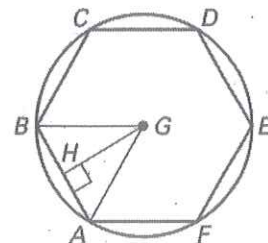
$$\frac{360}{n} = \frac{360}{6}$$

$$\frac{60}{2}$$

$90^\circ$

$$= 60^\circ$$

$$= 30^\circ$$



\* Note

$$\text{Perimeter} = P$$

$$P = (\text{side length})(\text{of sides})$$

$$P = s \cdot n$$

SUMMARY:

The area of a regular polygon can be found by

$$\text{Area} = \frac{1}{2} (\text{side length})(\text{apothem length})(\# \text{ of sides})$$

$$A = \frac{1}{2} (s)(a)(n)$$

$$\text{OR } A = \frac{1}{2} (\text{Perimeter})(a) = \frac{1}{2} P \cdot a$$

QUESTIONS:

**A3. Find the area of each regular polygon.**

a. Decagon with  $a \approx 5.7\text{cm}$  and  $s \approx 3.7\text{cm}$

$\hookrightarrow n=10$

$$A = \frac{1}{2} (3.7)(5.7)(10) \approx \boxed{105.45\text{cm}^2}$$

b. Octagon with  $a \approx 12.1\text{ft}$  and  $P \approx 80\text{ft}$

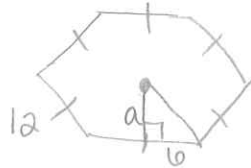
$\hookrightarrow n=8$

$$A = \frac{1}{2} P \cdot a = \frac{1}{2} (80)(12.1) = \boxed{484\text{ft}^2}$$

c. Hexagon with  $P = 72\text{ft}$

$\hookrightarrow n=6$

$$A = \frac{1}{2} P(a) = \frac{1}{2} (72)(10.39) \approx \boxed{374.12\text{ft}^2}$$

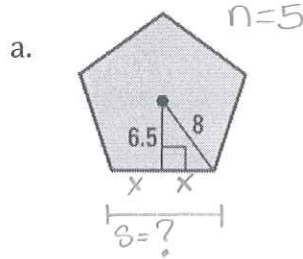


$\frac{72}{6} = 12'$  per side  
 $\frac{360}{6} = 60^\circ$



$\tan 30^\circ = \frac{6}{a}$   
 $a \approx 10.39$

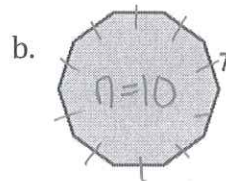
**A4. Find the perimeter and area of the regular polygon.**



$6.5^2 + x^2 = 8^2$   
 $x \approx \sqrt{21.75} \approx 4.66$

$s = 9.33$   $a = 6.5$   
 $A = \frac{1}{2} (9.33)(6.5)(5)$

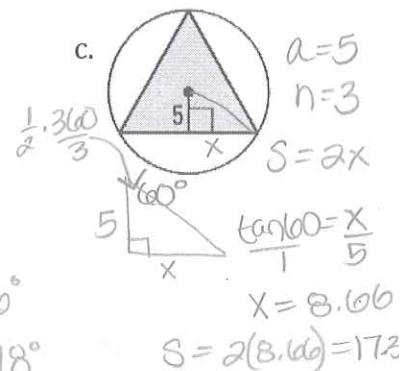
$A \approx \boxed{151.61\text{un}^2}$



$P = (10)(7) = 70$   
 $\frac{360}{10} = 36^\circ$   
 $36/2 = 18^\circ$

$\tan 18^\circ = \frac{3.5}{a}$   
 $a = 10.77$

$A = \frac{1}{2} P \cdot a = \frac{1}{2} (70)(10.77) = \boxed{376.95\text{un}^2}$



$A = \frac{1}{2} (17.32)(5)(3)$   
 $A = \boxed{129.9\text{un}^2}$

**A5. Find the side length of a regular 20-gon with an apothem of about 80 feet and an area of about 20,000 ft<sup>2</sup>.**

$n=20$   $a=80\text{ft}$   $A=20,000$   $s=?$

$A = \frac{1}{2} (s)(a)(n)$

$20,000 = \frac{1}{2} (s)(80)(20)$

$20,000 = (s)(800)$

$s = \boxed{25\text{ft}}$