Algebra III — Name ______________________________

Chapter 2 Note Packet

Section 2.1: Polynomial Functions
Essential Question:

Polynomials

-Have nonnegative exponents

-Variables ONLY in __________________

-General Form $a_n x^n + a_{n\ 1} x^{n\ 1} + \ldots + a_2 x^2 + a_1 x^1 + a_0$

Example 1

$27x^5 + 10x^4 \quad 6x^3 + 2x^2 \quad x + 7.5$

Terms: Coefficients:

- Always write in _________________ order
- **Leading Term** – Term with _________________ power
- **Leading coefficient** – coefficient of the leading term
- **Degree** of a polynomial – ______________ of the leading term

DEGREE	NAME	EXAMPLE
0		
1		
2		
3		
4		
5		

Example 2

Name the term of the polynomial $7x^4 + 10x^3 – 6x^2 + 5x + 11$

Constant: Linear Term: Quadratic Term:

Cubic Term: Quartic Term:

Reminder: Zeros of a function are where P(x) = 0 (set equation = 0)

Example 3

Decide if the function is a polynomial function and find the zeros.

A) f(x) = 7x - 5

B) $g(m) = m^3 - 25m$

C) $g(h) = h - \frac{1}{h}$

Example 4

Given $f(x) = \frac{x^2 - 7x + 12}{x + 5}$

a) Where is the function undefined?

b) Find the zeros of the function.

Example 5

Find the values of function $f(x) = 2x^2 + 15$.

a) f(3) =

b) f(3i) =

c) f(3n) =

d) f(n + 3) =

Example 6

$P(x) = 2x^4 - 3x^3 + x^2 + 6x - 6$

Find P(2) =

Another way to find P(x) is to use ____________________ ____________________

NOTE: You may only use synthetic substitution if x is a real number, x cannot be imaginary or a variable

LAST EXAMPLE ☹

Use synthetic substitution to find P(5) given: $f(x) = 6x^3 + 7x - 9$

Section 2.1 Summary:

Section 2.2: Synthetic Division, Remainder & Factor Theorem
Essential Question:

Review Long Division…

$$14\overline{)323}$$

Example 1

Is $(x + 4)$ a factor of $x^3 + x^2 - 10x + 8$?

Use synthetic division given $(x - a)$ the 'a' is the divisor

What is the a-value?

Example 2

Find $P(-4)$ given $P(x) = x^3 + x^2 - 10x + 8$

Example 3

Is (x - 2) a factor of $2x^2 - 7x + 1$?

Check (or think of this as another way):
Find P(2) =

Factor Theorem: For a polynomial P(x), x - a is a factor if and only if P(a) = 0

Remainder Theorem: When a polynomial P(x) is divided by x - a, the remainder is P(a)

Example 4

Find the remainder when $x^3 - 3x^2 + 5$ is divided by:

a) x - 3

b) x + 2

Example 5

Find the quotient and remainder of $(2x^3 + x^2 + 3x + 7)$ divided by (x + 2).

Example 6

Is x + 2 a factor of $x^{20} - 4x^{18} + 3x + 6$? Are you going to use synthetic division?

Example 7

Is x + 1 a factor of $P(x) = x^3 + 3x^2 + x - 1$?

Example 8

Given a polynomial equation and one or more roots find the remaining roots.

$P(x) = x^3 + 4x^2 + x - 6$ and $x = -2$

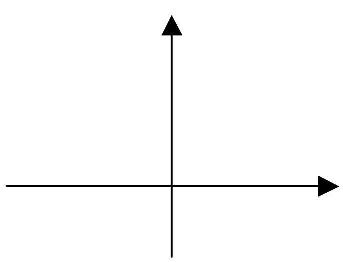
Section 2.2 Summary:

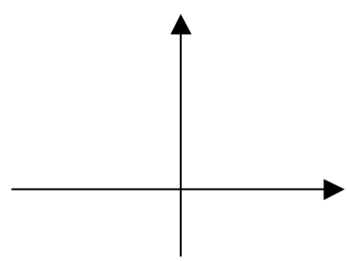
Section 2.3: Polynomial Equations

Essential Question:

CUBIC

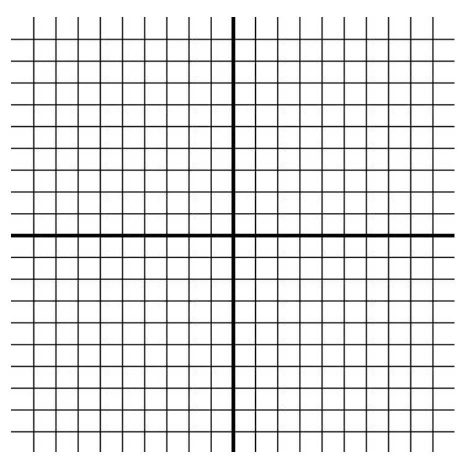
* $f(x) = ax^3 + bx^2 + cx + d$
* Shape is a sideways snake or ____ shape
* _______________ curve





Example 1

Sketch a graph of $f(x) = x(x - 2)(x + 3)$

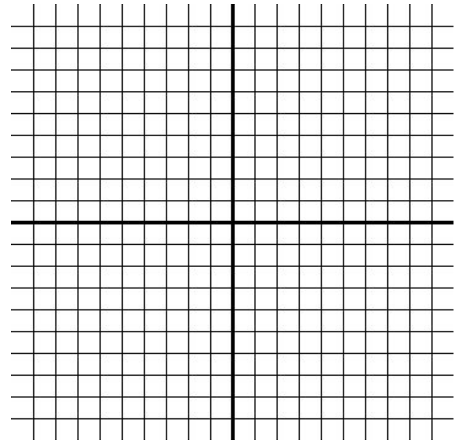


QUARTIC

* $f(x) = ax^4 + bx^3 + cx^2 + dx + e$
* Shape of graph is a _____ or a _____

Example 2

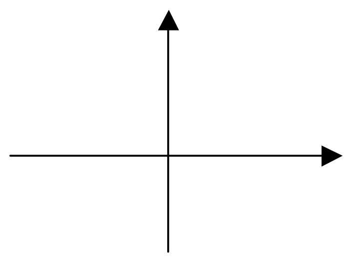
Graph $f(x) = (x + 3)(x + 1)(x - 1)(x - 2)$



Effect of a Squared or Cubed Term…

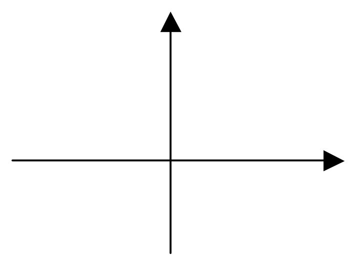
* If $P(x)$ has a squared term $(x - c)^2$
  then $x = c$ is a ____________ root

  -Graph is ____________ to the x-axis



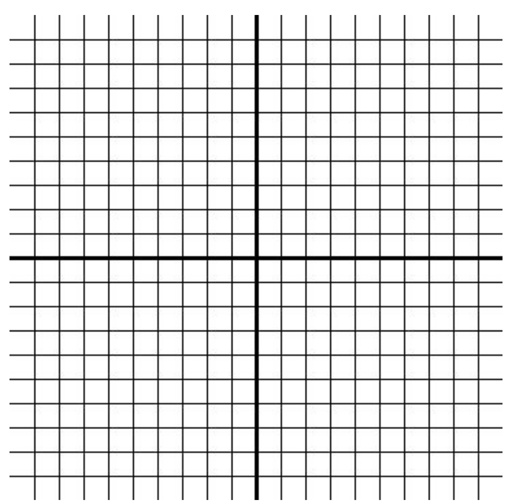
* If $P(x)$ has a cubed term $(x - c)^3$
  then $x = c$ is a _____________ root

  -Graph ________________ out around "c"
  and _______________ x-axis



Example 3

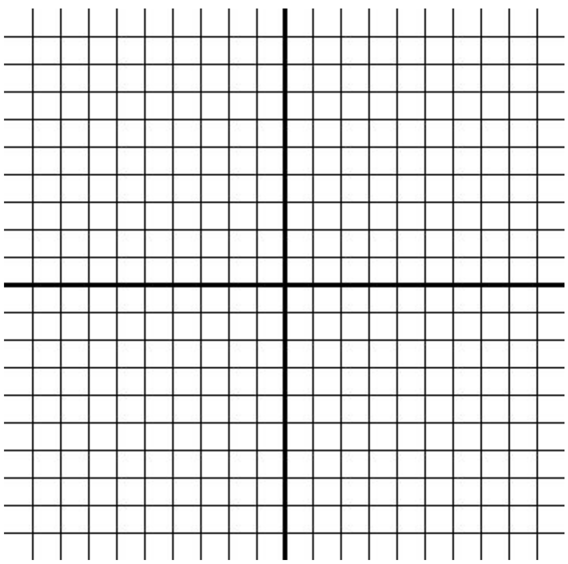
Graph $f(x) = (x - 1)^2(x + 4)$



Factor and graph each…

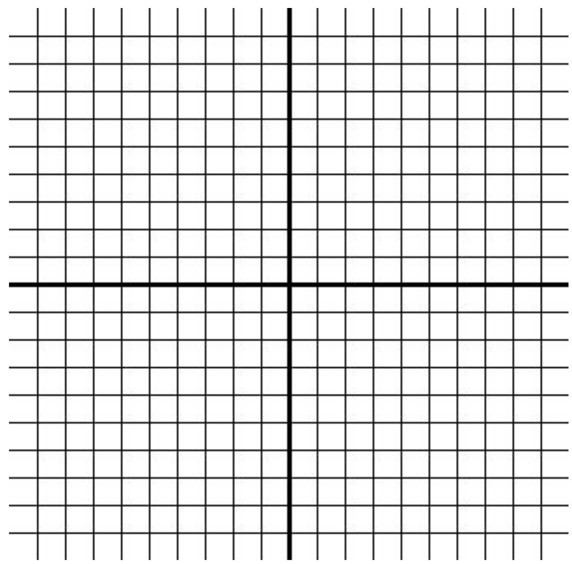
Example 4

$f(x) = x^4 - 9x^2$



Example 5

$f(x) = x^3 + 7x^2 + 12x$



	Quadratic	**Cubic**	**Quartic**
$a > 0$			
$a < 0$			

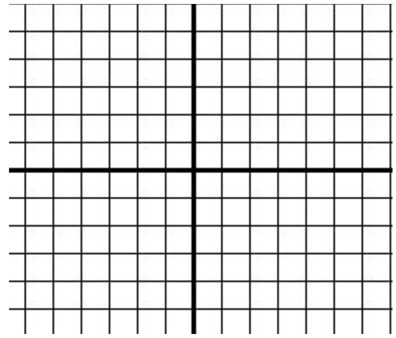
QUINTIC

$f(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$

If… $a > 0$ $a < 0$

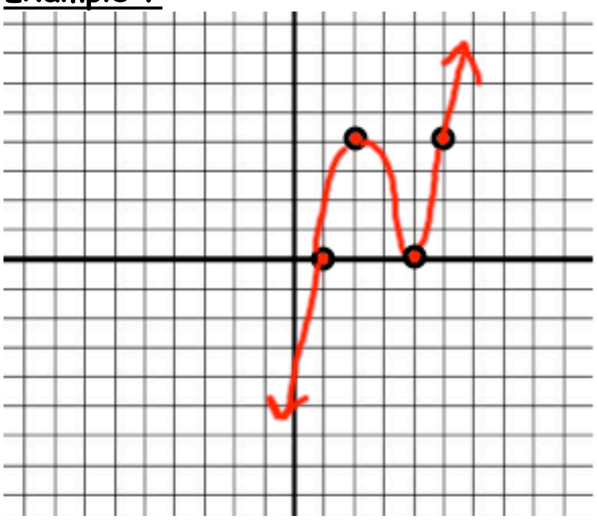
Example 6

Graph $f(x) = x^5 - 4x^3$

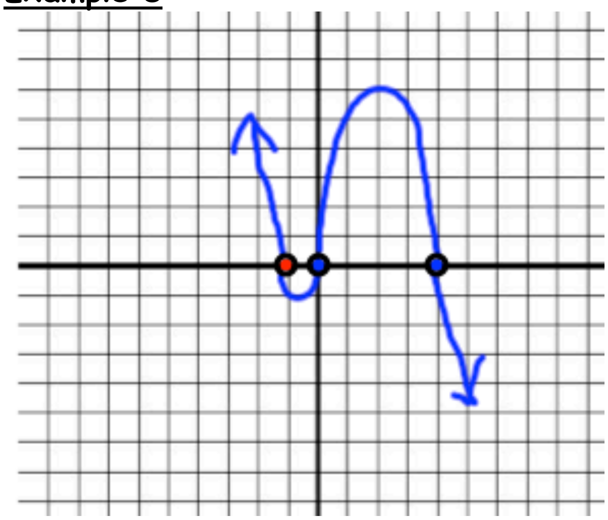


Directions: For the following examples name the type of equation and write an equation for the given graph.

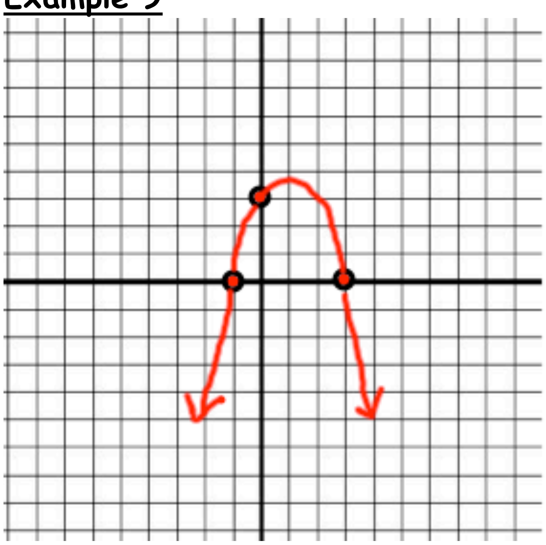
Example 7



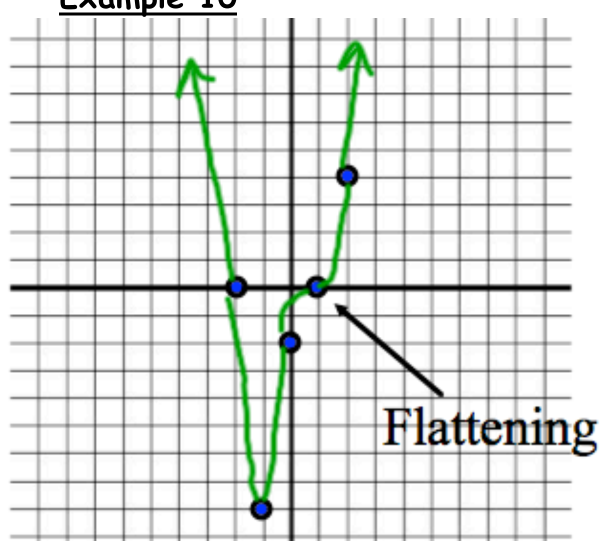
Example 8



Example 9



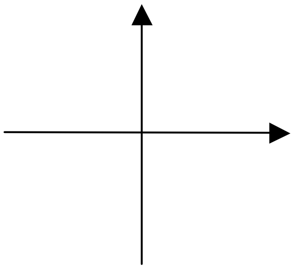
Example 10



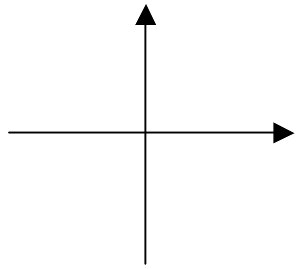
Example 11

Sketch a quartic equation with…

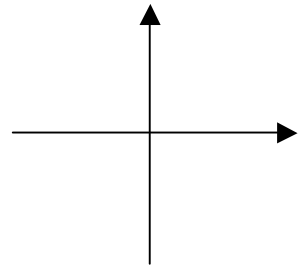
a) 3 zeros



b) 4 zeros



c) 1 zero

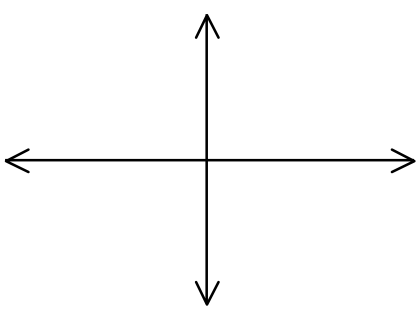


Section 2.3 Summary:

Section 2.4: Finding Maximums & Minimums of Polynomial Functions

Essential Question:

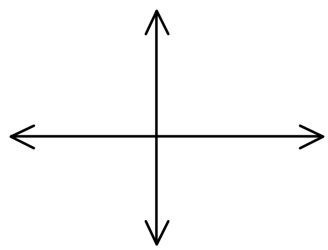
Let's Review QUADRATIC FUNCTIONS…



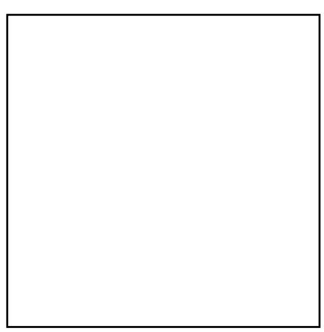
Example 1: Two numbers have a difference of 6. Find their minimum possible product.

Example 2: Your neighbor is constructing a dog pen using one side of his house as a wall for the pen. If he has 60 m of fencing for the other three sides, find the dimensions of the pen that give the greatest area.

CUBIC FUNCTIONS



Example 3: An open box is to be formed by cutting squares from a square sheet of metal 10 cm on a side and then folding up the sides. Find the approximate value of x that maximizes the volume. Then give the approximate maximum volume.



Section 2.4 Summary:

Section 2.6: Solve by Factoring
Essential Question:

1) Group

$x^3 + 4x^2 - 9x - 36 = 0$

2) GCF

$x^3 + 2x^2 - 3x = 0$

Review...

Factor $x^2 - 3x - 4 = 0$

3) Rewrite in Quadratic Form

a) $x^4 - 3x^2 - 4 = 0$

b) $2x^4 - x^2 - 3 = 0$

Rational Root Theorem

- Finds rational numbers that are ____________________ of polynomial equations
- $^p/_q$ is a solution where:
  - *p is a factor of the __________________
  - *q is a factor of the leading ____________________

4) Solve $3x^3 + 8x^2 - 9x + 2 = 0$

p:

q:

Section 2.6 Summary:

Section 2.7: Theorems of Polynomial Equations
Essential Question:

Fundamental Theorem of Algebra

If p(x) has degree n (n>0) with complex coefficients then p(x) = 0 has exactly n roots, where double roots count as 2, triple roots count as 3 and so on…

Complex Conjugates Theorem

If a + bi is a solution for p(x), so is ______________

If a + √b is a solution for p(x), so is ______________

Odd Degree

If p(x) has an odd degree then p(x) has at least __________ real root

Examples

Factor and find the following polynomial equations.

1) $x^3 + 6x^2 - 4x - 24 = 0$

2) $x^4 - 7x^2 + 12 = 0$

SUM and PRODUCT of ROOTS

Given a polynomial…

Sum of the roots $= \dfrac{-b}{a} = \dfrac{\text{coefficient of (n-1) term}}{\text{leading coefficient}}$

Product of the roots $= \dfrac{\text{constant}}{a}$ if the degree is ___________

$= \dfrac{-\text{constant}}{a}$ if the degree is __________

Examples

Find the sum and the product of the polynomial equations without factoring.

3) $x^2 + 5x + 6 = 0$

4) $x^4 - 11x^2 + 24 = 0$

5) $2x^3 + 5x^2 - 3x = 0$

Example 6

Find a quadratic equation whose roots are $3 \pm 5i$

Example 7

Find a cubic equation that has roots $(1 \quad \sqrt{6})$ and $\frac{3}{2}$.

Section 2.7 Summary: