

Chapter 2: Polynomial Functions

Section 1

Polynomials

- Have nonnegative exponents
- Variables **ONLY** in numerator
- General Form

$$a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x^1 + a_0$$

Example 1

$$27x^5 + 10x^4 - 6x^3 + 2x^2 - x + 7.5$$

Terms:

$$\begin{array}{ll} 27x^5 & 10x^4 \\ -6x^3 & 2x^2 \\ -x & 7.5 \end{array}$$

Coefficients:

$$\begin{array}{l} 27, 10, -6, 2, \\ -1, \text{ and } 7.5 \end{array}$$

- Always write in descending order
- Leading Term - Term with highest power $27x^5$
- Leading coefficient - coefficient of the leading term 27
- Degree of a polynomial - power of the leading term 5

Alg III 2.1 lesson

<u>DEGREE</u>	<u>NAME</u>	<u>EXAMPLE</u>
0	constant	7
1	Linear	7x
2	Quadratic	$4x^2 + 7$
3	Cubic	$x^3 + 6x - 4$
4	Quartic	$5x^4 - 6x^3 + x$
5	Quintic	$18x^5 + x^3 - x - 4$

Example 2

Name the term of the polynomial $7x^4 + 10x^3 - 6x^2 + 5x + 11$

Constant: 11 Linear Term: 5x

Quadratic Term: $-6x^2$ Cubic Term: $10x^3$

Quartic Term: $7x^4$

Alg III 2.1 lesson

Zeros of a function are where $P(x) = 0$

(set equation = 0)

zeros → where graph crosses x-axis

Example 3

Decide if the function is a polynomial function and find the zeros.

A) $f(x) = 7x - 5$

yes

$$7x - 5 = 0$$

$$+5 \quad +5$$

$$\frac{7x}{7} = \frac{5}{7}$$

$$\boxed{x = \frac{5}{7}}$$

B) $g(m) = m^3 - 25m$

$$g(m) = m^3 - 25m$$

yes

$$m^3 - 25m = 0$$

$$m(m^2 - 25) = 0$$

Diff. of squares

$$m(m+5)(m-5) = 0$$

$$\boxed{m = 0, -5, 5}$$

C) $g(h) = h - \frac{1}{h}$

Not poly.

$$h \left(h - \frac{1}{h} \right) = 0$$

$$h^2 - 1 = 0$$

$$\sqrt{h^2} = \sqrt{1}$$

$$\boxed{h = \pm 1}$$

Example 4

Given $f(x) = \frac{x^2 - 7x + 12}{x + 5}$

- a) Where is the function undefined?

when denominator = 0

$$x + 5 = 0$$

if $x = -5$

- b) Find the zeros of the function.

$$\frac{0}{\text{any}\#} = 0 \quad \therefore \text{set numerator} = 0$$

$$x^2 - 7x + 12 = 0$$

$$(x-3)(x-4)$$

$$\boxed{x=3} \quad \boxed{x=4}$$

x-intercepts

Example 5

Find the values of function $f(x) = 2x^2 + 15$.

a) $f(3) = 2(3)^2 + 15 = 2(9) + 15 = 18 + 15 = 33$

b) $f(3i) = 2(3i)^2 + 15 = 2(9i^2) + 15 = 18i^2 + 15$

c) $f(3n) = 2(3n)^2 + 15 = 2(9n^2) + 15 - 18 + 15 = \boxed{-3}$

d) $f(n + 3) = \boxed{18n^2 + 15}$

$$2(n+3)^2 + 15$$

FOIL

$$2(n^2 + 6n + 9) + 15$$

$$2n^2 + 12n + 18 + 15 = \boxed{2n^2 + 12n + 33}$$

Example 6

$$P(x) = 2x^4 - 3x^3 + x^2 + 6x - 6$$

Find $P(2) = \boxed{18}$

Another way to find $P(x)$ is to use **SYNTHETIC SUBSTITUTION**

NOTE: You may only use synthetic substitution if x is a real number, x cannot be imaginary or a variable

$$\begin{array}{r} 2 \\ \underline{-} \quad 2 \quad -3 \quad 1 \quad 6 \quad -6 \\ \downarrow \quad 4 \quad 2 \quad 6 \quad 24 \\ \hline 2 \quad 1 \quad 3 \quad 12 \quad \boxed{18} = P(2) \end{array}$$

LAST EXAMPLE 😢

Use synthetic substitution to find P(5) given

$$f(x) = 6x^3 + 7x - 9$$

$$\begin{array}{r} 5 | \quad 6 \quad 0 \quad 7 \quad -9 \\ \downarrow \qquad \qquad \qquad \qquad \\ \hline 6 \quad 30 \quad 150 \quad 785 \\ \hline 6 \quad 30 \quad 157 \quad \textcircled{776} = P(5) \end{array}$$

