Section 2.2
Synthetic Division: The Remainder & Factor Theorem

Review long division

$$\begin{array}{r} 23 \quad \text{Quotient} \\ \text{Divisor } 14\overline{)323} \quad \text{Dividend} \\ \underline{-28} \\ 43 \\ \underline{-42} \\ 1 \quad \text{Remainder} \end{array}$$

Check:

$$14(23) = 322$$
$$\underline{+\ 1}$$
$$323$$

Conclude: (quotient x divisor) + remainder = dividend

Is 14 a factor of 323? NO $\text{Rem} \neq 0$

If the remainder is zero then the divisor is a factor.

Example 1

Is (x + 4) a factor of $x^3 + x^2 - 10x + 8$?

Use synthetic division given (x - a) the 'a' is the divisor

What is the a-value? $(x+4) = (x - -4)$ $a = -4$

$$\begin{array}{r|rrrr} -4 & 1 & 1 & -10 & 8 \\ & \downarrow & -4 & 12 & -8 \\ \hline & 1 & -3 & 2 & 0 = \text{remainder} \end{array}$$

$\boxed{\text{yes, because } R = 0}$

Example 2

Find P(-4) given $P(x) = x^3 + x^2 - 10x + 8$

$P(-4) = -64 + 16 + 40 + 8 = 0$

plug-n-chug

Example 3

Is (x - 2) a factor of $2x^2 - 7x + 1$?

$(x-a)$

$$\begin{array}{r|rrr} 2 & 2 & -7 & 1 \\ & \downarrow & 4 & -6 \\ \hline & 2 & -3 & \boxed{-5=R} \end{array} \quad \therefore (x-2) \text{ NOT A factor}$$

$R \neq 0$

Check (or think of this as another way):

Find P(2) = $2(2)^2 - 7(2) + 1$

'a' value $\quad 2(4) - 14 + 1 = -5 \neq 0$

Factor Theorem

For a polynomial P(x), x - a is a factor if and only if P(a) = 0

Remainder Theorem

When a polynomial P(x) is divided by x - a, the remainder is P(a)

Example 4

Find the remainder when $x^3 - 3x^2 + 5$ is divided by

a) x - 3

$$\begin{array}{r|rrrr} 3 & 1 & -3 & 0 & 5 \\ & \downarrow & 3 & 0 & 0 \\ \hline & 1 & 0 & 0 & 5 \end{array}$$

$\boxed{R=5}$

b) x + 2

$P(-2) = (-2)^3 - 3(-2)^2 + 5$

$$\begin{array}{r|rrrr} -2 & 1 & -3 & 0 & 5 \\ & & -2 & 10 & -20 \\ \hline & 1 & -5 & 10 & -15 \end{array}$$

$-8 - 3(4) + 5$

$-8 - 12 + 5 = -15$

$\boxed{R=-15}$

Example 5

Find the quotient and remainder

$2x^3 + x^2 + 3x + 7 \div x + 2$

$$\begin{array}{r|rrrr} -2 & 2 & 1 & 3 & 7 \\ & \downarrow & -4 & 6 & -18 \\ \hline & 2 & -3 & 9 & -11 \end{array}$$

Quotient

$\boxed{2x^2 - 3x + 9}$

Remainder $= \boxed{-11}$

Example 6

Is x + 2 a factor of $x^{20} - 4x^{18} + 3x + 6$?

Are you going to use synthetic division?

No because there are a lot of missing terms.

$x+2$

$a = -2$

$P(-2) = (-2)^{20} - 4(-2)^{18} + 3(-2) + 6$

$= 0$

$\boxed{\text{yes, } x+2 \text{ is a factor because } R=0.}$

Example 7

Is x + 1 a factor of $P(x) = x^3 + 3x^2 + x - 1$?

$a = -1$

$$
\begin{array}{r|rrrr}
-1 & 1 & 3 & 1 & -1 \\
 & \downarrow & -1 & -2 & 1 \\
\hline
 & 1 & 2 & -1 & 0
\end{array}
$$

$\boxed{R=0}$ so yes x+1 is a factor

$Q = x^2 + 2x - 1$

Example 8

Given a polynomial equation and one or more roots find the remaining roots.

$P(x) = x^3 + 4x^2 + x - 6$ and $x = -2$ comes from the factor $(x+2)$

a-value

$$
\begin{array}{r|rrrr}
-2 & 1 & 4 & 1 & -6 \\
 & \downarrow & -2 & -4 & 6 \\
\hline
 & 1 & 2 & -3 & 0=R
\end{array}
$$

$Q = x^2 + 2x - 3$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$\boxed{x=-3}$ $\boxed{x=1}$

Check by multiplying the factors.

$\underbrace{(x + 2)(x + 3)}_{x^2+5x+6}(x - 1) = (x-1)(x^2+5x+6) = x^3 + 5x^2 + 6x$

$-x^2 - 5x - 6$

$x^3 + 4x^2 + x - 6$ ✓

Homework

p61 #1 - 19 odds