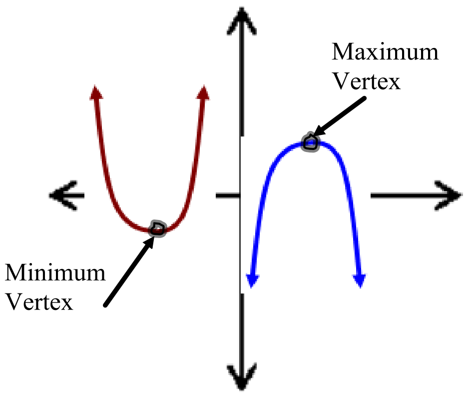
Finding MAXIMUMS and MINIMUMS of Polynomial Functions

QUADRATIC FUNCTIONS



$f(x) = ax^2 + bx + c$ and $a \neq 0$

A.O.S $x = -b \div 2a$

To get the vertex plug the A.O.S value into the function and solve for y

vertex is (-b/2a , max or min value)

Example 1
Two numbers have a difference of 6.
Find their minimum possible product.

1st number = x

2nd number = $x+6$

Product = multiplication
Thus multiply the
1st & 2nd number $x(x+6)$

x^2+6x

a>0 open up
minimum
vertex

$x = \frac{-b}{2a} = \frac{-6}{2(1)} = \frac{-6}{2} = -3$

plug the x-value 3
into the equation to find y

$f(-3) = (-3)^2 + 6(-3)$
$= 9 + -18 = -9$

$\boxed{\text{min product} = -9}$

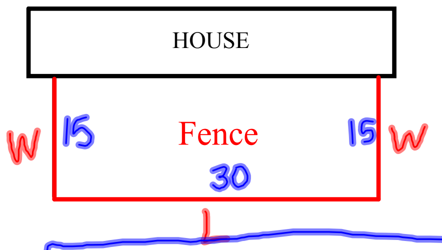
$-3 \cdot 3 = -9$ -3 & 3 have a difference of 6.

OR instead of defining x(x+6)
you could have used x(x - 6)
thus $f(x) = x^2 - 6x$
$-b/2a = 6/2 = 3$
$f(3) = 3^2 - 6(3) = 9 - 18 = -9$
you still get the **min = -9**

Example 2

Your neighbor is constructing a dog pen using one side of his house as a wall for the pen.

If he has 60 meters of fencing for the other three sides, find the dimensions of the pen that give the greatest area.



Max the area

$A = L \cdot W = (60-2w)w$

$P = 2W + L$

$60 = 2W + L$

$-2w \quad -2w$

$60 - 2w = L$

plug into Area formula

$A = 60w - 2w^2$

a<0 opens down

$\frac{-b}{2a} = \frac{-60}{2(-2)} = \frac{-60}{-4}$

$x = 15$

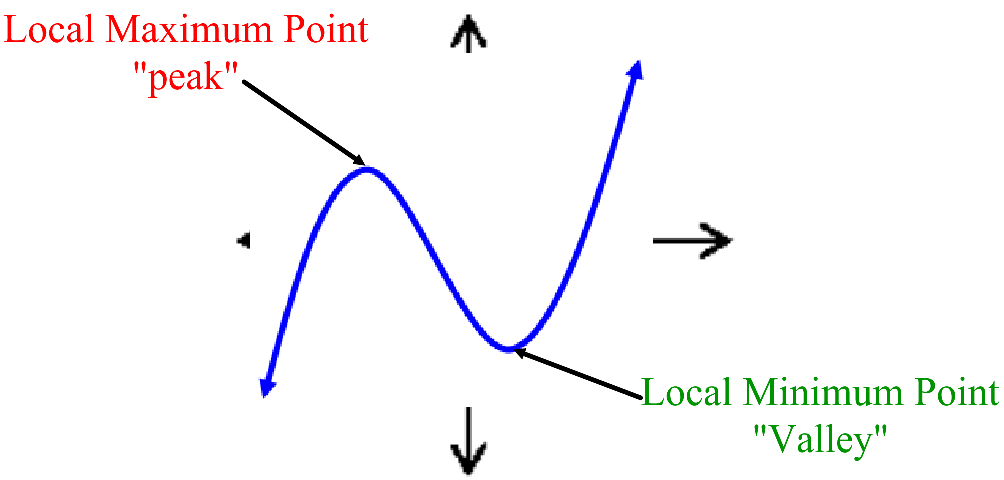
$f(15) = 60(15) - 2(15)^2$

$= 450$

$\boxed{\text{max area} = 450\ m^2}$

$(15)(30) = 450$

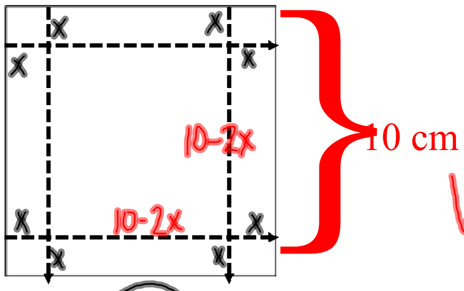
CUBIC FUNCTIONS



Example 3

An open box is to be formed by cutting squares from a square sheet of metal 10 cm on a side and then folding up the sides.

Find the approximate value of x that maximizes the volume. Then give the approximate maximum volume.



$V = L \cdot W \cdot H$

$V = (10-2x)(10-2x)x$

$V = x(10-2x)^2$ *cubic

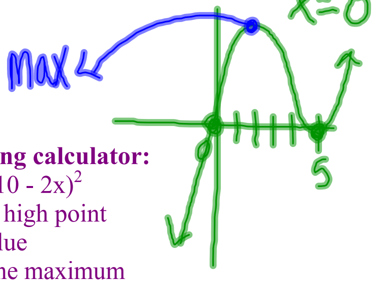
$x = 0$ $\quad$ $10 - 2x = 0$

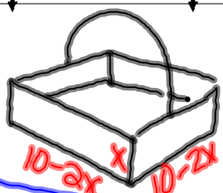
$10 = 2x$

$\frac{10}{2} = \frac{2x}{2}$

$x = 5$

Double Tangent





Max Volume $= 74.05 \text{ cm}^3$

Using a graphing calculator:
Graph $f(x) = x(10 - 2x)^2$
Find the "peak" high point
Locate the y-value
The y-value is the maximum

HW

p70 $\quad$ CE #1-4
WE #1, 3, 10