Section 1.5: Complex Numbers

Essential Question:
What is a complex number?
When is a complex number simplified?

Real Numbers
Represented by a number line
Divided into rational numbers and irrational numbers
Every number is a real number!

Complex Numbers
Have the form $a+bi$

* remember $i=\sqrt{-1}$, $i^2=-1$

a = real part b = imaginary part i = imaginary unit

Ex: $5+3i$ or $-7+i\sqrt{5}$

Pure Imaginary Numbers
When a = 0 (no real part)

Ex: $8i$ or $i\sqrt{2}$

Examples:
Rewrite as a complex number.

1) $\sqrt{-36}$
$\sqrt{-1}\cdot\sqrt{36}$
$i\cdot 6$
$6i$

2) $\sqrt{-15}$
$\sqrt{-1}\sqrt{15}$
$i\sqrt{15}$

3) $\sqrt{-3}\sqrt{-6}$
$i\sqrt{3}\cdot i\sqrt{6}$
$i^2\sqrt{18}=(-1)\sqrt{9}\sqrt{2}$
$=(-1)(3)\sqrt{2}$
$=-3\sqrt{2}$

Simplify.

4) $(6-5i)+(9+7i)$
$15+2i$

5) $(7+3i)(4-i)$ FOIL
$28-7i+12i-3i^2$ $i^2=-1$
$28+5i+3=31+5i$

6) $(8-5i)(8+5i)$ *conjugates
$64+40i-40i-25i^2$
outside & inside terms will cancel
$64+25=89$

7) $(8-5i)+(8+5i)$
16

Conjugates
In the form $a+bi$ and $a-bi$
Sum is a real number
Product is nonnegative real number

* To be simplified → NO imaginary unit in denominator

Examples:

8) $\frac{1}{3-4i}\cdot\frac{(3+4i)}{(3+4i)}$ outside/inside will cancel

*multiply by the conjugate

$= \frac{3+4i}{9-16i^2}$

$= \frac{3+4i}{9+16} = \boxed{\frac{3+4i}{25}}$

9) $\frac{8}{i}\cdot\frac{(-i)}{(-i)} = \frac{-8i}{-1i^2}$

$= \frac{-8i}{-1(-1)} = \frac{-8i}{+1} = -8i$

10) $\frac{(5+2i)}{(3+i)}\cdot\frac{(3-i)}{(3-i)} = \frac{15-5i+6i-2i^2}{9-i^2}$

$\frac{15+1i+2}{9+1} = \boxed{\frac{17+i}{10}}$

11) $i^2 = \boxed{-1}$

$i^3 = i^2\cdot i = -1\cdot i = \boxed{-i}$

$i^4 = i^3\cdot i = -i\cdot i = -i^2 = \boxed{+1}$

$i^5 = i^4\cdot i = 1(i) = \boxed{i}$

$i^6 = i^5\cdot i = i\cdot i = i^2 = \boxed{-1}$

$i^7 = i^6\cdot i = -1\cdot i = \boxed{-i}$

12) i^{-4}

$= \frac{1}{i^4} = \frac{1}{1} = \boxed{1}$

Equal Complex Numbers

a + bi and c + di are equal if…

a = c and b = d

Ex13) Find the value of x and y.

$2x + y + (3 - 5x)i = 1 - 7i$

a b c d

2nd plug in

$2x+y=1$

$2(2)+y=1$

$4+y=1$

$\boxed{y=-3}$

1st

$3-5x=-7$

$-5x=-10$

$\boxed{x=2}$

Section 1.5 Summary:

A complex number has a real part (a # on the # line) and an imaginary part (contains i). Form is: $a \pm bi$

A complex # is simplified when the imaginary unit has an exponent of one (i) and no imaginary unit in the denominator of a fraction. If "i" exists in the denominator, multiply by the conjugate.