Section 1.6: Quadratic Equations

Essential Question:

How do you solve a quadratic equation?

Quadratic Equations

- Standard form $ax^2+bx+c=0$
- Solutions are called zeros or roots (where graph crosses x-axis) x-intercepts
- 3 methods to solve quadratic equations…

1) Factor
2) Complete the square
3) Quadratic Formula

Examples

Factor. and solve.

1) $x^2 - 5x - 14 = 0$
   a b c

a=1 → factor right away

a·c: -14; $-7 \times +2$; -5 = b

$(x-7)(x+2)=0$

$x-7=0$ $\quad x+2=0$

$x=7$ $\quad x=-2$

$(7,0)$ $\quad (-2,0)$

2) $16m^2 - 24m = 0$

2 terms → GCF

$8m(2m-3)=0$

$\frac{8m}{8}=\frac{0}{8}$ $\quad 2m-3=0$

$m=0$ $\quad 2m=3$

$m=3/2$

$(0,0)$ $\quad (3/2,0)$

3) $(3x-2)(x+4)=-11$

FOIL

$\neq 0$

$3x^2+12x-2x-8=-11$

$+11 \quad +11$

1st get =0

$3x^2+10x+3=0$
a b c

2nd Find new x-terms since a≠1

a·c: 9; $+1x \times +9x$; 10 = b

a≠1 so new x-terms

3rd group

$3x^2+1x+9x+3=0$

$x(3x+1)+3(3x+1)$

common factor ✓

$(3x+1)(x+3)=0$

$x=-1/3$ $\quad x=-3$

$(-1/3,0)$ $\quad (-3,0)$

Section 1.6: How To Complete The Square

Step 1: Given $ax^2 + bx + c = 0$ Move 'c' to the other side of the equation
The equation should now look like the following…

$x^2 + bx + \square = c + \square$

Step 2: The a-value must be 1 Divide every term by a-value
If the a-value is NOT 1 then factor the 'a' value

Step 3: Now complete the square $\square$

You do so by $\left(\frac{b}{2}\right)^2$ Now your equation should look like …

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$$

Step 4: Factor the left side of the equation

You will go from $x^2 + bx + \left(\frac{b}{2}\right)^2$ to $\left(x+\frac{b}{2}\right)^2$

These are the same thing! Check by FOILing $\left(x+\frac{b}{2}\right)^2$

$$=\left(x+\frac{b}{2}\right)\left(x+\frac{b}{2}\right) = x^2 + 2x\left(\frac{b}{2}\right) + \left(\frac{b}{2}\right)^2 = x^2 + bx + \left(\frac{b}{2}\right)^2$$

Step 5: Now to solve for x take the square root of both sides

$$\left(x+\frac{b}{2}\right)^2 = c+\left(\frac{b}{2}\right)^2 \longrightarrow \sqrt{\left(x+\frac{b}{2}\right)^2} = \sqrt{c+\left(\frac{b}{2}\right)^2} \longrightarrow x+\frac{b}{2} = \pm\sqrt{c+\left(\frac{b}{2}\right)^2}$$

Here the square^2 and $\sqrt{\ }$ cancel

Step 6: Use basic algebra to get 'x' by itself

EXAMPLE

4) $x^2 - 10x + 14 = 0$

Step1 $x^2 - 10x \mid -14 = -14$ (−14 | −14)

Step2 $a = 1$ ✓

Step3 $\left(\frac{b}{2}\right)^2 = \left(\frac{-10}{2}\right)^2 = (-5)^2 = 25$ add to both sides

$x^2 - 10x + 25 = -14 + 25$

Step4 Factor $(x-5)^2 = 11$

Step5 $\sqrt{(x-5)^2} = \pm\sqrt{11}$

Step6 $x - 5 = \pm\sqrt{11}$
$+5 \quad +5$

$x = 5 \pm \sqrt{11}$

Solve by completing the square:

5) $m^2 + 8m = -30$

Step1

Step2 $a=1$

$m^2+8m+16 = -30+16$

Step3 $\left(\frac{b}{2}\right)^2=\left(\frac{8}{2}\right)^2=(4)^2=16$

Step4 $(m+4)^2 = -14$

Step5 $\sqrt{(m+4)^2} = \pm i\sqrt{14}$

Step6 $m+4 = \pm i\sqrt{14}$

$m = -4 \pm i\sqrt{14}$

6) $4w^2 - 8w - 32 = 0$

Step1 $\frac{4w^2-8w}{4} = \frac{32}{4}$ $(\div 4)$

Step2 $w^2-2w = 8$

Step3 $\left(\frac{-2}{2}\right)^2=(-1)^2=1$ $w^2-2w+1 = 8+1$

Step4 $(w-1)^2 = 9$

Step5 $\sqrt{(w-1)^2} = \pm\sqrt{9}$

Step6 $w-1 = \pm 3$

$w = 1 \pm 3 \rightarrow 4 = w$ or $-2 = w$

Quadratic Formula

$$X = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

*Finding *intercepts when $ax^2 + bx + c = 0$

Examples

Solve using the quadratic formula.

7) $4x^2 - x - 7 = 0$

$a=4$, $b=-1$, $c=-7$

$$x = \frac{1 \pm \sqrt{(-1)^2-4(4)(-7)}}{2(4)}$$

$$= \frac{1 \pm \sqrt{1+112}}{8} = \frac{1 \pm \sqrt{113}}{8}$$

8) $y^2 + 10y + 35 = 0$

$a=1$, $b=10$, $c=35$

$$\frac{-10 \pm \sqrt{(10)^2-4(1)(35)}}{2(1)}$$

$$\frac{-10 \pm \sqrt{100-140}}{2} = \frac{-10 \pm \sqrt{-40}}{2}$$

$$= \frac{-10 \pm i\sqrt{4}\sqrt{10}}{2} = \frac{-10 \pm 2i\sqrt{10}}{2} = -5 \pm i\sqrt{10}$$

9) $m^2 = 10m - 25$

$m^2-10m+25=0$

$a=1$, $b=-10$, $c=25$

$$\frac{10 \pm \sqrt{(-10)^2-4(1)(25)}}{2(1)} = \frac{10 \pm \sqrt{100-100}}{2}$$

$$= \frac{10 \pm \sqrt{0}}{2} = \frac{10 \pm 0}{2} = \frac{10}{2} = 5 = x$$

Double Root ($x=5$ twice)

10) $8x^2 = 7 - 10x$

$8x^2+10x-7=0$

$$\frac{-10 \pm \sqrt{(10)^2-4(8)(-7)}}{2(8)}$$

$$\frac{-10 \pm \sqrt{100+224}}{16}$$

$$\frac{-10 \pm \sqrt{324}}{16}$$

$\frac{-10 \pm 18}{16}$

$\frac{8}{16}$ or $\frac{-28}{16}$

$x = \frac{1}{2}$ $x = \frac{-7}{4}$

Discriminant

The quantity beneath the $\sqrt{\ }$ = b^2-4ac

Determines if solutions are real or imaginary

Value of Discriminant	Nature of Roots
$b^2 - 4ac = 0$	1 real root (vertex on x-axis)
$b^2 - 4ac < 0$	2 imaginary roots (no x-intercepts)
$b^2 - 4ac > 0$	2 real roots (2 x-intercepts)

Be careful… do NOT cancel binomials by dividing ➔ loose a zero/root

11) $2(x-3) = (x-3)^2 \rightarrow (x-3)(x-3)$ FOIL

$2x-6 = x^2-6x+9$

$-2x\ +6 \qquad -2x\ +6$

$0 = x^2-8x+15 \longrightarrow (x-5)(x-3)$

$x=5$

$x=3$

Be Careful… when you square both sides of an equation ➔ gain an extraneous answer

12) $\left(\sqrt{3x+6}\right)^2 = (2x+1)^2 \rightarrow (2x+1)(2x+1)$ FOIL

$3x+6 = 4x^2+4x+1$

$-3x\ -6 \qquad -3x\ -6$

$0 = 4x^2+1x-5 \longrightarrow (4x+5)(x-1)$

$x = -5/4$ extraneous

$x=1$

Check: $\sqrt{3(1)+6} \stackrel{?}{=} 2(1)+1$

$3 = 3$ ✓

$\sqrt{3(-5/4)+6} \stackrel{?}{\neq} 2(-5/4)+1$

Be careful… when you have denominators ➔ check if answer makes den=0 if so then extraneous!

13) $(x-2)(x+2)\left(\dfrac{x+2}{x-2}+\dfrac{x-2}{x+2}=\dfrac{8-4x}{x^2-4}\right)$

$(x+2)(x+2)+(x-2)(x-2) = 8-4x$

$x^2+4x+4+x^2-4x+4 = 8-4x$

$2x^2+4x = 0$

GCF $2x(x+2)=0$

$2x=0$ → $x=0$

$x+2=0$ → $x=-2$ extraneous → makes den=0

Section 1.6 Summary:

① Factor: try GCF, if $a\neq 1$ → $a\cdot c$, b → new x-terms then group

② Complete the Square

Step1: isolate constant

Step2: Divide everything by a-value

Step3: Take $(b/2)^2$ and add to both sides

Step4: Factor $x^2+x+(b/2)$ side as $\left(x+\frac{b}{2}\right)^2$

Step5: $\sqrt{\ }$ both sides

Step6: solve for x

③ Quadratic Formula

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

Simplify! Reduce!