

## Section 1.6: Quadratic Equations

Essential Question:

How do you solve a quadratic equation?

### Quadratic Equations

- Standard form  $ax^2 + bx + c = 0$
- Solutions are called zeros or roots (where graph crosses x-axis)
- 3 methods to solve quadratic equations... x-intercepts

- 1) Factor
- 2) Complete the square
- 3) Quadratic Formula

### Examples

Factor and solve.

1)  $|x^2 - 5x - 14 = 0$   
 $\begin{matrix} a & b & c \end{matrix}$

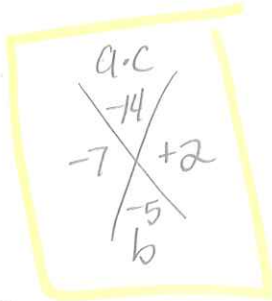
$a=1 \rightarrow$  factor right away

$$(x-7)(x+2) = 0$$

$$x-7=0 \quad x+2=0$$

$$x=7 \quad x=-2$$

$$\boxed{(7,0)} \quad \boxed{(-2,0)}$$



2)  $16m^2 - 24m = 0$

2 terms  $\rightarrow$  GCF

$$8m(2m-3) = 0$$

$$\frac{8m}{8} = 0 \quad \frac{2m-3}{2} = 0$$

$$m=0 \quad m=3/2$$

$$\boxed{(0,0)} \quad \boxed{(3/2,0)}$$

3)  $(3x-2)(x+4) = -11$

FOIL  $\neq 0$

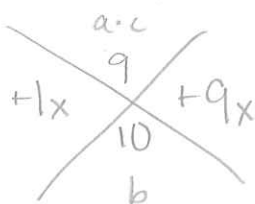
$$3x^2 + 12x - 2x - 8 = -11$$

$$3x^2 + 10x + 3 = 0$$

$$\begin{matrix} a & b & c \end{matrix}$$

1st get = 0

2nd Find new x-terms since  $a \neq 1$



$a \neq 1$   
 so new x-terms

$$3x^2 + 1x + 9x + 3 = 0$$

3rd group

$$x(3x+1) + 3(3x+1)$$

common factor  $\checkmark$

$$(3x+1)(x+3) = 0$$

$$x = -1/3 \quad x = -3$$

$$\boxed{(-1/3,0)} \quad \boxed{(-3,0)}$$

## Section 1.6: How To Complete The Square

**Step 1:** Given  $ax^2 + bx + c = 0$  Move 'c' to the other side of the equation  
The equation should now look like the following...

$$x^2 + bx + \square = c + \square$$

**Step 2:** The a-value must be 1  
If the a-value is NOT 1 then factor the 'a' value

*Divide every term by a-value*

**Step 3:** Now complete the square  

You do so by  $\left(\frac{b}{2}\right)^2$  Now your equation should look like ...

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$$

**Step 4:** Factor the left side of the equation

You will go from  $x^2 + bx + \left(\frac{b}{2}\right)^2$  to  $\left(x + \frac{b}{2}\right)^2$

These are the same thing! Check by FOILING  $\left(x + \frac{b}{2}\right)^2$

$$= \left(x + \frac{b}{2}\right) \left(x + \frac{b}{2}\right) = x^2 + 2x\left(\frac{b}{2}\right) + \left(\frac{b}{2}\right)^2 = x^2 + bx + \left(\frac{b}{2}\right)^2$$

**Step 5:** Now to solve for x take the square root of both sides

$$\left(x + \frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2 \longrightarrow \sqrt{\left(x + \frac{b}{2}\right)^2} = \sqrt{c + \left(\frac{b}{2}\right)^2} \longrightarrow x + \frac{b}{2} = \pm \sqrt{c + \left(\frac{b}{2}\right)^2}$$

Here the square<sup>2</sup> and  $\sqrt{\quad}$  cancel

**Step 6:** Use basic algebra to get 'x' by itself

### EXAMPLE

4)  $x^2 - 10x + 14 = 0$

Step 1  $x^2 - 10x = -14$

Step 2  $a=1 \checkmark$

Step 3  $\left(\frac{b}{2}\right)^2 = \left(\frac{-10}{2}\right)^2 = (-5)^2 = 25$  add to both sides

$x^2 - 10x + 25 = -14 + 25$

Step 4 Factor  $(x-5)^2 = 11$

Step 5  $\sqrt{(x-5)^2} = \pm\sqrt{11}$

Step 6  $x-5 = \pm\sqrt{11}$

$x = 5 \pm \sqrt{11}$

Solve by completing the square:

5)  $m^2 + 8m = -30$  ← Step 1

Step 2  
 $a=1$  →  $m^2 + 8m + 16 = -30 + 16$

Step 3  
 $\left(\frac{b}{2}\right)^2 = \left(\frac{8}{2}\right)^2 = (4)^2 = 16$

Step 4  $(m+4)^2 = -14$

Step 5  $\sqrt{(m+4)^2} = \pm i\sqrt{14}$

Step 6  $m+4 = \pm i\sqrt{14}$

$m = -4 \pm i\sqrt{14}$

**Quadratic Formula**

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

\* Finding x-intercepts

when  $ax^2 + bx + c = 0$

**Examples**

Solve using the quadratic formula.

7)  $4x^2 - x - 7 = 0$

$a=4$   
 $b=-1$   
 $c=-7$   
 $x = \frac{1 \pm \sqrt{(-1)^2 - 4(4)(-7)}}{2(4)}$

$= \frac{1 \pm \sqrt{1+112}}{8} = \frac{1 \pm \sqrt{113}}{8}$

8)  $y^2 + 10y + 35 = 0$

$a=1$   
 $b=10$   
 $c=35$

$\frac{-10 \pm \sqrt{(10)^2 - 4(1)(35)}}{2(1)}$

$= \frac{-10 \pm \sqrt{100 - 140}}{2} = \frac{-10 \pm \sqrt{-40}}{2}$

$= \frac{-10 \pm i\sqrt{40}}{2} = \frac{-10 \pm 2i\sqrt{10}}{2} = -5 \pm i\sqrt{10}$

9)  $m^2 = 10m - 25$

$m^2 - 10m + 25 = 0$

$a=1$   
 $b=-10$   
 $c=25$

$\frac{10 \pm \sqrt{(-10)^2 - 4(1)(25)}}{2(1)} = \frac{10 \pm \sqrt{100 - 100}}{2}$

$= \frac{10 \pm \sqrt{0}}{2} = \frac{10 \pm 0}{2} = \frac{10}{2} = 5 = x$   
 Double Root  
 (x=5 twice)

10)  $8x^2 = 7 - 10x$

$8x^2 + 10x - 7 = 0$

$\frac{-10 \pm \sqrt{(10)^2 - 4(8)(-7)}}{2(8)} = \frac{-10 \pm 18}{16}$

$\frac{-10 \pm \sqrt{100 + 224}}{16}$

$\frac{-10 \pm \sqrt{324}}{16}$

$\frac{8}{16}$  or  $-\frac{28}{16}$

$x = \frac{1}{2}$     $x = -\frac{7}{4}$

## Discriminant

The quantity beneath the  $\sqrt{\quad} = \underline{b^2 - 4ac}$

Determines if solutions are real or imaginary

Value of Discriminant

$$b^2 - 4ac = 0$$

Nature of Roots

1 real root (vertex on x-axis)

$$b^2 - 4ac < 0$$

2 imaginary roots (no x-intercepts)

$$b^2 - 4ac > 0$$

2 real roots (2 x-intercepts)

Be careful... do NOT cancel binomials by dividing  $\rightarrow$  lose a zero/root

11)  $2(x-3) = (x-3)^2 \rightarrow (x-3)(x-3)$  FOIL

$$\begin{array}{r} 2x-6 = x^2-6x+9 \\ -2x+6 \quad -2x+6 \end{array}$$

$$x=5$$

$$x=3$$

$$0 = x^2 - 8x + 15 \rightarrow (x-5)(x-3)$$

Be Careful... when you square both sides of an equation  $\rightarrow$  gain an extraneous answer

12)  $(\sqrt{3x+6})^2 = (2x+1)^2 \rightarrow (2x+1)(2x+1)$  FOIL

$$\begin{array}{r} 3x+6 = 4x^2+4x+1 \\ -3x-6 \quad -3x-6 \end{array}$$

$$0 = 4x^2 + 1x - 5$$

$$(4x+5)(x-1)$$
  
$$x = \cancel{5/4} \quad \boxed{x=1}$$
  
extraneous

Check:  $\frac{\sqrt{3(1)+6}}{3} \stackrel{?}{=} 2(1)+1$   
 $3 = 3 \checkmark$

$$\frac{\sqrt{3(\cancel{5/4})+6}}{3} \neq 2(\cancel{5/4})+1$$

Be careful... when you have denominators  $\rightarrow$  check if answer makes den=0  
if so then extraneous!

13)  $(x^2)(x+2) \left( \frac{x+2}{x-2} + \frac{x-2}{x+2} = \frac{8-4x}{x^2-4} \right)$

$$(x+2)(x+2) + (x-2)(x-2) = 8-4x$$

$$x^2 + \cancel{4x} + 4 + x^2 - \cancel{4x} + 4 = 8-4x$$

$$2x^2 + 4x = 0$$

$$\text{GCF } 2x(x+2) = 0$$

$$2x=0$$

$$\boxed{x=0}$$

$$x+2=0$$

$$x = \cancel{-2} \rightarrow \text{makes den}=0$$
  
extraneous

### Section 1.6 Summary:

① Factor: try GCF, if  $a \neq 1 \rightarrow \frac{a/c}{b} \frac{\#}{\#} \leftarrow$  new x-terms then group

② Complete the Square

step 1: isolate constant

step 2: Divide everything by a-value

step 3: Take  $(b/2)^2$  and add to both sides

step 4: Factor  $x^2 + x + (b/2)^2$  side as  $(x + \frac{b}{2})^2$

step 5:  $\sqrt{\quad}$  both sides

step 6: solve for x

③ Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Simplify! Reduce!