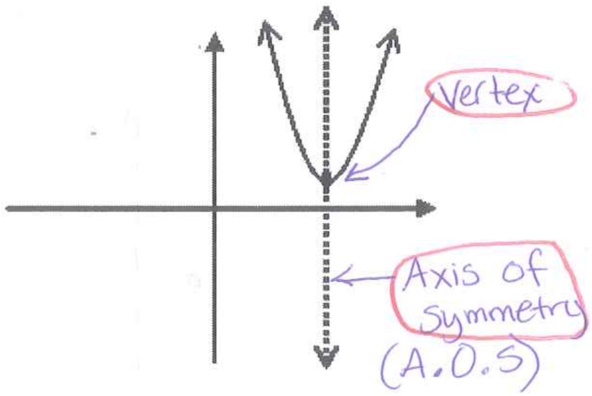
Section 1.7: Quadratic Functions and their Graphs

Essential Question: How do you graph a quadratic function?

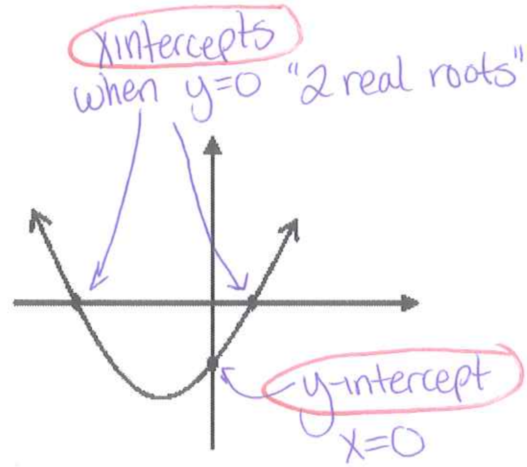
-Form is $f(x) = ax^2 + bx + c,\ a \neq 0$
-Graph satisfies $y = ax^2 + bx + c$
-Graph is called a Parabola

Labels/Terminology

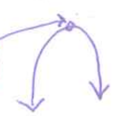


$X = \frac{-b}{2a}$

Intercepts



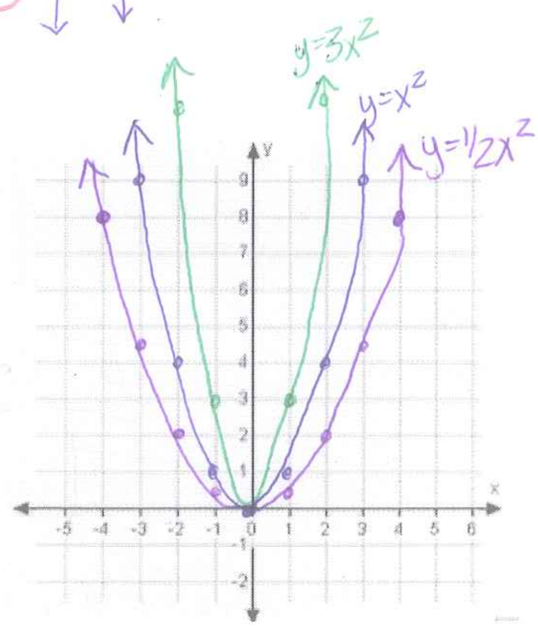
a-value	**Opening**	**Vertex**
$a > 0$	up	Minimum
$a < 0$	down	Maximum



Parent Graph: $\boxed{y = x^2}$

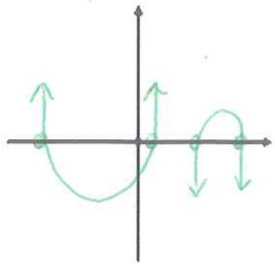
$\boxed{y = 3x^2}$

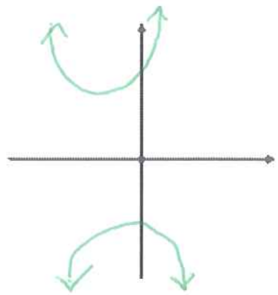
$\boxed{y = \frac{1}{2}x^2}$

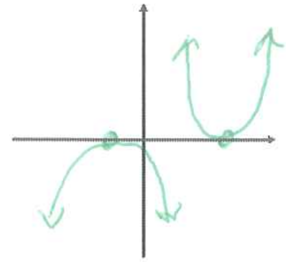


Note: The bigger |a-value| the more narrower the parabola
*increases more rapidly

Number of Roots







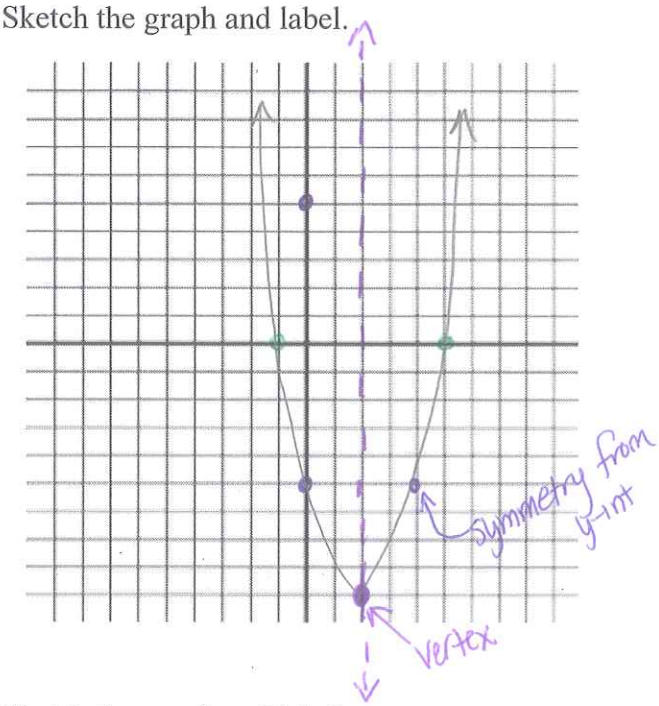
2 real roots (2 xint) | 2 imaginary roots (No xint) | 1 real root (1 x-int = vertex)

$b^2 - 4ac > 0$ | $b^2 - 4ac < 0$ | $b^2 - 4ac = 0$ * known as a double root

Example 1

Find the intercepts, A.O.S., and vertex of the parabola. Sketch the graph and label.

$$y = x^2 - 4x - 5$$



x-intercepts

$y=0$ $\quad 0 = x^2 - 4x - 5$
$(x-5)(x+1)$ $\quad x=5 \quad x=-1$ $\quad \boxed{(5,0)\ (-1,0)}$

y-intercepts

$x=0$ $\quad y = 0^2 - 4(0) - 5$
$y = -5$ $\quad \boxed{(0,-5)}$

*Note yint = constant

AOS

$x = \frac{-b}{2a} = \frac{4}{2(1)} = 2$ $\quad \boxed{x=2}$

Vertex

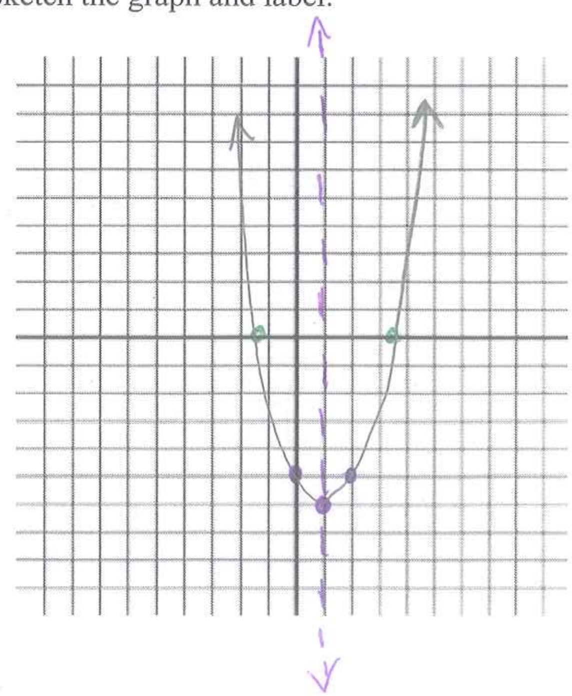
on A.O.S ∴ $x=2$

$y = (2)^2 - 4(2) - 5$
$4 - 8 - 5$
$y = -9$ $\quad \boxed{(2,-9)}$

Example 2

Find the intercepts, A.O.S., and vertex of the parabola. Sketch the graph and label.

$$y = x^2 - 2x - 5$$



x-intercepts

$y=0$ $\quad 0 = x^2 - 2x - 5$

$\frac{2 \pm \sqrt{(-2)^2 - 4(1)(-5)}}{2(1)} = \frac{2 \pm \sqrt{24}}{2}$ $\quad \boxed{x = 3.4 \quad x = -1.4}$

y-intercepts

$x=0$ $\quad y = 0 - 0 - 5$
$y = -5$ $\quad \boxed{(0,-5)}$

AOS

$x = \frac{-b}{2a} = \frac{2}{2(1)} = 1$ $\quad \boxed{x=1}$

Vertex

$x=1$ $\quad y = (1)^2 - 2(1) - 5$
$1 - 2 - 5$
$y = -6$ $\quad \boxed{(1,-6)}$

Another Way To Graph: $y = a(x - h)^2 + k$ Vertex: (h, k)
opposite
A.O.S: $x = h$

Example 3

Graph and label: $y = x^2 + 4x + 9$ * lets make the equation in (h,k) form

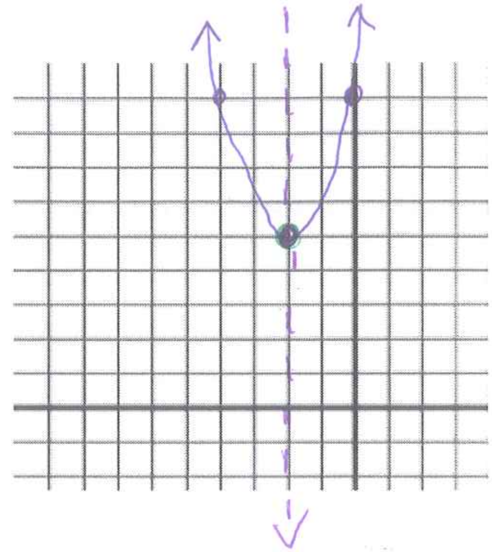
* complete square

$\left(\frac{4}{2}\right)^2 = (2)^2 = 4$

$x^2 + 4x \boxed{+4} = -9 \boxed{+4}$

$(x+2)^2 = -5$

$y = (x+2)^2 + 5$



Vertex $\boxed{(-2, 5)}$

AOS $\boxed{x = -2}$

y-intercepts

$x = 0$ constant $= 9$ $\boxed{(0, 9)}$

x-intercepts

Solve $(x+2)^2 = -5$

$\sqrt{(x+2)^2} = \pm\sqrt{-5}$

$= \pm i\sqrt{5}$ *imaginary → No x-int.

This is expected because we know ↑↗ opens up since a-value positive and vertex is above the x-axis

Methods of Graphing Parabolas

Method #1: AOS: $x = \frac{-b}{2a}$ and Vertex: $\left(\frac{-b}{2a}, y\right)$

(plug in x-value & solve for y-value)

Method #2: $y = a(x - h)^2 + K$ Vertex: (h, k)

AOS: $x = h$

*Note: $y = ax^2 + bx + c$ the y-intercept is always the c-value

Section 1.7 Summary:

1st Find the xint by setting equation $= 0$ then either 1) Factor 2) Complete square or 3) Quadratic Formula.

2nd Find the yint by plugging $x = 0$; y-int = constant $(0, c)$

3rd Find the A.O.S by $x = -b/2a$, then plug this into equation & solve for y to get vertex $\left(\frac{-b}{2a}, y\right)$ 4th

*Plot all of these pieces to graph the parabola!