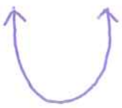
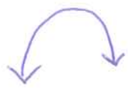
Section 1.8: Quadratic Models

Essential Question:

How do quadratic equations model real-world situations?

When to use a quadratic model:

*Values decrease and then increase  parabola opens up

*Values increase and then decrease  parabola opens down

* V = $h \cdot \pi r^2$ SA = $2\pi r^2 + 2\pi r \cdot h$

(cylinder)

EXAMPLE #1

Use the given values to find an equation of the form $f(x) = ax^2 + bx + c$.

$f(1) = 4$, $f(2) = 12$, $f(4) = 46$

$4 = a(1)^2 + b(1) + c$

$4 = a + b + c$

* cancel a, b, or c

(mult by -1)

$12 = a(2)^2 + b(2) + c$

$12 = 4a + 2b + c$

$-4 = -a - b - c$

$(8 = 3a + b)(-3)$

Now cancel another coeff.

$46 = a(4)^2 + b(4) + c$

$46 = 16a + 4b + c$

$-4 = -a - b - c$

$42 = 15a + 3b$

$-24 = -9a - 3b$

$18 = 6a$

$a = 3$

use in previous equation

$8 = 3(3) + b$

$8 = 9 + b$

$b = -1$

plug in a=3 & b=-1 into original equation

$4 = a + b + c$

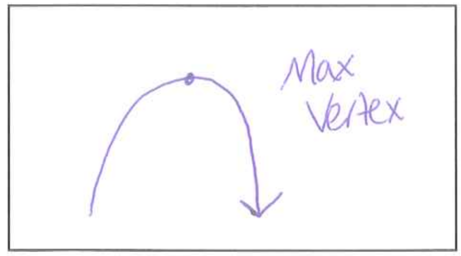
$4 = 3 + -1 + c$

$4 = 2 + c$

$c = 2$

$$\boxed{f(x) = \underset{a}{3}x^2 + \underset{b}{-1}x + \underset{c}{2}}$$

EXAMPLE #2



Sketch of parabola

In an electric circuit, the available power P in watts when a current of I amperes is flowing is given by $P = 110I - 11I^2$. $a=-11$ $b=110$

If you were to plot coordinates (I , P)

(amp , power watts)

a. If the current is increased from 2 amperes to 3 amperes, by how much will the power increase?

$P(2) = 110(2) - 11(2)^2$
$= 220 - 44$
$P(2) = 176$

$P(3) = 110(3) - 11(3)^2$
$= 330 - 99$
$P(3) = 231$

$231 - 176$

$\boxed{55 \text{ watts}}$

b. Find the maximum power that can be produced by the circuit.

Max @ vertex

$\left(\frac{-b}{2a}, y\right)$

$x = \frac{-b}{2a} = \frac{-110}{2(-11)} = \frac{110}{22} = 5$ amps

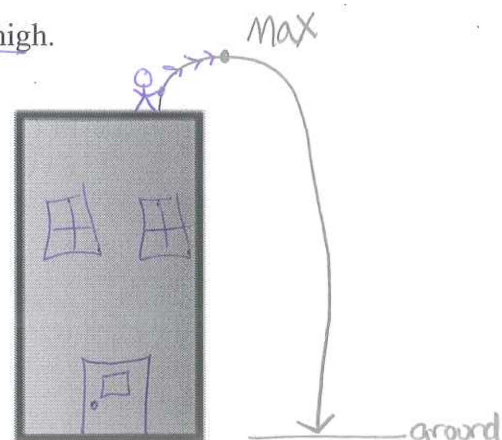
$P(5) = 110(5) - 11(5)^2$
$= 550 - 275 = 275$

$\boxed{\text{Max Power is } 275 \text{ watts}}$

EXAMPLE #3

An object thrown into the air with an initial velocity (v_0) meters per second from a height (h_0) meters above ground is modeled by the function $h(t) = -4.9t^2 + v_0t + h_0$ (model does not account for air resistance). The height of the object will be h(t) after t seconds.

A ball is tossed with an upward velocity of 16 m/s from a building 20m high.



a. Find its height above the ground t seconds later.

$h(t) = -4.9t^2 + 16t + 20$

b. When will the ball reach its highest elevation?

@ the max vertex

time?

$x = \frac{-b}{2a} = \frac{-16}{2(-4.9)} = \boxed{1.63 \text{ sec.}}$

If plug in then finding highest elevation (height)

c. When will it hit the ground?

When on ground height $= 0 = y$

$0 = -4.9t^2 + 16t + 20$
a b c

$\frac{-16 \pm \sqrt{(16)^2 - 4(-4.9)(20)}}{2(-4.9)} = \frac{-16 \pm \sqrt{256 + 392}}{-9.8}$

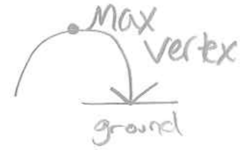
$\frac{-16 \pm \sqrt{648}}{-9.8} = \frac{-16 \pm 25.46}{-9.8}$

$t = -.96$ cannot have neg. time

$\boxed{t = 4.23 \text{ sec.}}$

Section 1.8 Summary:

If you were to throw an object in the air you could find the maximum height by finding the vertex. You could also find the time it hits the ground using the model $h(\text{time}) = -4.9t^2 + (\text{velocity})t + (\text{height @ } t=0)$



If an object fell and bounced you could find the time it took to hit the ground.