Algebra III
Chapter 2
Review

Name KEY

Date ____________

1) Let $f(x) = x^2 + x$. Evaluate and simplify each of the following.

a. $f(3i)$

$(3i)^2 + (3i)$

$9i^2 + 3i$

$\boxed{-9 + 3i}$

b. $f(n + 2)$

$(n+2)^2 + (n+2)$

FOIL

$\hookrightarrow n^2 + 4n + 4 + n + 2$

$\boxed{n^2 + 5n + 6}$

2) Find the quotient and the remainder when $x^4 - 2x^2 - 10$ is divided by $x - 1$. $= 0$

$x = 1$

$$\begin{array}{r|rrrrr} 1 & 1 & 0 & -2 & 0 & -10 \\ & \downarrow & 1 & 1 & -1 & -1 \\ \hline & 1 & 1 & -1 & -1 & -11 = R \end{array}$$

$\boxed{Q = x^3 + x^2 - x - 1}$

$\boxed{R = -11}$

3) Is $x + 2$ a factor of $x^4 - 2x^3 + 9x - 8 = 0$? Explain your decision.

$x = -2$

$$\begin{array}{r|rrrrr} -2 & 1 & -2 & 0 & 9 & -8 \\ & \downarrow & -2 & 8 & -16 & 14 \\ \hline & 1 & -4 & 8 & -7 & 6 = R \end{array}$$

Not a factor

because remainder $\neq 0$ it $= 6$

4) Find the sum and the product of the following.

a. $2x^4 + 4x^3 - 6x^2 + 2x - 4 = 0$

Sum $= \frac{-b}{a} = \frac{-4}{2} = \boxed{-2}$

product $= \frac{\text{const}}{a} = \frac{-4}{2} = \boxed{-2}$

odd deg.

b. $x^3 + 2x - 5 = 0$

Sum $= \frac{-b}{a} = \frac{0}{1} = \boxed{0}$

product $= \frac{-\text{const}}{a} = \frac{5}{1} = \boxed{5}$

5) Write the polynomial function that has a degree of 3, a constant of 5, a linear coefficient of -4, whose quadratic term is $7x^2$, and a leading coefficient of 9.

cubic $\rightarrow ax^3 + bx^2 + cx + d$

$\boxed{9x^3 + 7x^2 - 4x + 5}$

6) Find all of the zeros of $p(x) = x^3 + 3x^2 - 4x - 12$.

$x^2(x+3) - 4(x+3)$

$(x+3)(x^2-4)$

$(x+3)(x+2)(x-2)$

$x = -3$

$x = -2$

$x = 2$

$\nearrow$ 2 roots

7) Find a quadratic equation with integral coefficients and with the roots of $4 \pm \sqrt{3}$.

$$\text{sum} = 4+\sqrt{3} + 4-\sqrt{3} = \frac{8}{1} = \frac{-b}{a} \qquad a=1 \quad b=-8$$

$$\text{product} = (4+\sqrt{3})(4-\sqrt{3})$$

$$= 16-3 = 13 = \frac{\text{constant}}{a} \qquad c=13$$

$$\boxed{y = x^2 - 8x + 13}$$

8) A given polynomial function has roots of 2, and $5 \pm i$. Based on this information, describe the graph of this polynomial function.

3 roots means cubic function, crossing at 2 on the x-axis and snake or "S" shape

9) Find a cubic equation with integral coefficients and with roots 3 and 5 + i.

$\hookrightarrow x = 5 \pm i$

$\rightarrow (x-3)$ factor

$$\text{sum} = 5+i + 5-i = 10 = \frac{-b}{a} \Rightarrow \begin{matrix} a=1 \\ b=-10 \end{matrix}$$

$$\text{product} = (5+i)(5-i) = 25-i^2 = 25+1 = 26 = c$$

$$(x-3)(x^2-10x+26) = x^3 - 10x^2 + 26x - 3x^2 + 30x - 78$$

$$\boxed{f(x) = x^3 - 13x^2 + 56x - 78}$$

10) If x = 3 is a root of $2x^3 - 5x^2 - 4x + 3 = 0$, find the remaining roots.

$$\begin{array}{r|rrrr} 3 & 2 & -5 & -4 & 3 \\ & \downarrow & 6 & 3 & -3 \\ \hline & 2 & 1 & -1 & 0=R \end{array}$$

$$Q = 2x^2 + x - 1$$

$$(2x-1)(x+1)$$

$$2x-1=0 \qquad x+1=0$$

$$\boxed{x = \frac{1}{2}} \qquad \boxed{x = -1}$$

11) The polynomial $p(x) = 5x^4 - 3x + 2$ is not factorable. List all the possible rational roots to this quartic function. You do NOT have to solve the function.

(constant) $p: \pm 1 \quad \pm 2$

(L.C.) $q = \pm 1 \quad \pm 5$

$$\boxed{p/q = \pm 1 \ \pm 2 \ \pm 1/5 \ \pm 2/5}$$

Solve the following equations by factoring. Give all real and imaginary roots.

12) $\underbrace{x^3 - 3x^2}_{} + \underbrace{4x - 12}_{} = 0$

$$x^2(x-3) + 4(x-3)$$

$$(x-3)(x^2+4) = 0$$

$$x-3=0 \qquad x^2+4=0 \qquad x = \pm\sqrt{-4}$$

$$\boxed{x=3} \qquad x^2 = -4 \qquad \boxed{x = \pm 2i}$$

13) $x^4 + 3x^2 - 10$

-10; 5, -2; 3

$$(x^2+5)(x^2-2)$$

$$x^2 = -5 \qquad x^2 = 2$$

$$x = \pm\sqrt{-5} \qquad \boxed{x = \pm\sqrt{2}}$$

$$\boxed{x = \pm i\sqrt{5}}$$

14) Sketch the graph of the following.

$y = x^4 - 10x^3 + 25x^2$

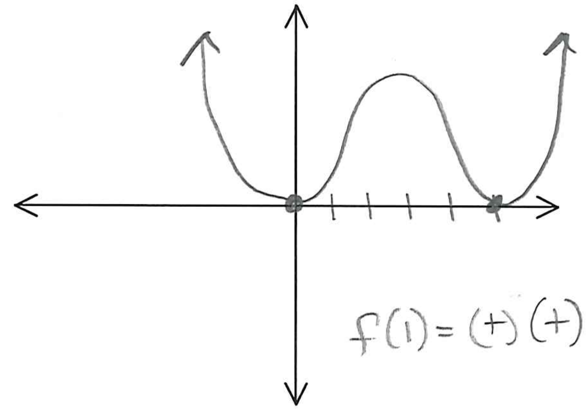
$a=1 \rightarrow$ pos

$x^2(x^2-10x+25)$

$x^2(x-5)(x-5)$

$x^2(x-5)^2$

$x=0 \quad x=5$ Both Double Roots $\hookrightarrow$ tangent



$f(1) = (+)(+) = \text{pos}\uparrow$

15) Let $p(x) = 7 - 8x + x^2$. Does this graph have a maximum or minimum value? Explain your decision. Also, find the maximum or minimum value.

$p(x) = x^2 - 8x + 7$

$a=1 \rightarrow$ pos

a is positive so has a minimum

Vertex (x, y)

$\hookrightarrow x = \frac{-b}{2a}$

$x = \frac{8}{2(1)} = 4$

$f(4) = (4)^2 - 8(4) + 7 = 16 - 32 + 7 = -9$

Min $= -9$

16) Name the type of equation, leading term, leading coefficient, degree, and constant of the following polynomials.

a) $8x^3 + x^2 - 6x + 5$

Name: Cubic

$LT = 8x^3$ $\quad D = 3$

$LC = 8$ $\quad$ const $= 5$

b) $8x^4 - 7x + 3 - x^5$

Name: Quintic

$LT = -x^5$ $\quad D = 5$

$LC = -1$ $\quad$ const $= 3$

17) Given $2x^3 - 7x + 2 = 0$

3 roots

a) List all of the possible rational roots of the function.

$p: \pm 1 \ \pm 2$

$q: \pm 1 \ \pm 2$

$p/q = \pm 1 \ \pm 1/2 \ \pm 2$

b) Find the roots of the function.

	2	0	-7	2
1	2	2	-5	$-3 \neq 0$
-1	2	-2	-5	$7 \neq 0$
2	2	4	1	$4 \neq 0$
$\boxed{x = -2}$	2	-4	1	0 = Rem

1 root

use quotient to find other 2 roots

$Q = 2x^2 - 4x + 1$ (a, b, c)

$$\frac{4 \pm \sqrt{(-4)^2 - 4(2)(1)}}{2(2)}$$

$$\frac{4 \pm \sqrt{16-8}}{4} = \frac{4 \pm \sqrt{8}}{4}$$

$$\frac{4 \pm 2\sqrt{2}}{4}$$

$$\boxed{x = \frac{2 \pm \sqrt{2}}{2}}$$

2 roots