Section 2.2: Synthetic Division, Remainder & Factor Theorem

Essential Question:

What is synthetic division used for?

Review Long Division...

$$\begin{array}{r} 23 \leftarrow \text{Quotient} \\ 14\overline{)323} \leftarrow \text{Dividend} \\ -28\downarrow \\ \hline 43 \\ -42 \\ \hline 1 = \text{Remainder} \end{array}$$

Divisor →

Check:

$(14)(23) + (1) = 323$

$322 + 1 = 323$ ✓

* (Divisor)(Quotient) + Remainder = Dividend

Example 1

Is $(x + 4)$ a factor of $x^3 + x^2 - 10x + 8$?

Use synthetic division given $(x - a)$ the 'a' is the divisor

What is the a-value? $x+4=0$ $x=-4$

$$\begin{array}{r|rrrr} -4 & 1 & 1 & -10 & 8 \\ & & -4 & 12 & -8 \\ \hline & 1 & -3 & 2 & 0 = \text{remainder} \end{array}$$

yes, because remainder = 0

Example 2

Find P(-4) given $P(x) = x^3 + x^2 - 10x + 8$

$P(-4) = (-4)^3 + (-4)^2 - 10(-4) + 8 = 0$ ← remainder

Example 3

Is $(x - 2)$ a factor of $2x^2 - 7x + 1$?

$x-2=0$

$x=2$

$$\begin{array}{r|rrr} 2 & 2 & -7 & 1 \\ & & 4\downarrow & -6 \\ \hline & 2 & -3 & -5 \neq 0 \end{array}$$

NO, remainder ≠ 0

Check (or think of this as another way):

Find P(2) = $2(2)^2 - 7(2) + 1$

$8 - 14 + 1 = -5 \neq 0$

Factor Theorem: For a polynomial P(x), x - a is a factor if and only if P(a) = 0

Remainder Theorem: When a polynomial P(x) is divided by x - a, the remainder is P(a)

Example 4

Find the remainder when $x^3 - 3x^2 + 5$ is divided by:

a) $x - 3$

$x=3$

$$\begin{array}{r|rrr} 3 & 1 & -3 & 0 & 5 \\ & & 3\downarrow & 0 & 0 \\ \hline & 1 & 0 & 0 & 5 = R \end{array}$$

b) $x + 2$ $x=-2$

$$\begin{array}{r|rrr} -2 & 1 & -3 & 0 & 5 \\ & & -2\downarrow & 10 & -20 \\ \hline & 1 & -5 & 10 & -15 = R \end{array}$$

OR $P(-2) = (-2)^3 - 3(-2)^2 + 5 = -15$

Example 5

Find the quotient and remainder of $(2x^3 + x^2 + 3x + 7)$ divided by $(x + 2)$.

$x = -2$

$\frac{x^3}{x} = x^2$ (quadratic)

$-2 \mid 2 \quad 1 \quad 3 \quad 7$

$\quad\quad\downarrow \quad -4 \quad 6 \quad -18$

$\quad\quad 2 \quad -3 \quad 9 \quad -11$

Quotient $= 2x^2 - 3x + 9$ | Rem $= -11$

Example 6

Is $x + 2$ a factor of $x^{20} - 4x^{18} + 3x + 6$? Are you going to use synthetic division? NO

$x = -2$

Degree $= 20$ missing a lot of terms between 20 and 0

NO, use calculator or mental math

$P(-2) = (-2)^{20} - 4(-2)^{18} + 3(-2) + 6 = 0$ ✓

yes, $(x+2)$ is a factor because remainder $= 0$

Example 7

Is $x + 1$ a factor of $P(x) = x^3 + 3x^2 + x - 1$?

$x = -1$

$-1 \mid 1 \quad 3 \quad 1 \quad -1$

$\quad\quad\downarrow \quad -1 \quad -2 \quad 1$

$\quad\quad 1 \quad 2 \quad -1 \quad 0 = R$

yes

Quotient $= x^2 + 2x - 1$

Rem $= 0$

Example 8

Given a polynomial equation and one or more roots find the remaining roots.

$P(x) = x^3 + 4x^2 + x - 6$ and $x = -2$ comes from $(x+2)$

$-2 \mid 1 \quad 4 \quad 1 \quad -6$

$\quad\quad\downarrow \quad -2 \quad -4 \quad 6$

$\quad\quad 1 \quad 2 \quad -3 \quad 0 = R$

Quotient; expect because -2 is a zero

Quotient $= x^2 + 2x - 3 = 0$

$(x+3)(x-1)$

$x = -3$ | $x = 1$ (plug into f(x) and = zero)

Check: $(x+2)(x^2+2x-3) = x^3 + 4x^2 + x - 6$ ✓

Section 2.2 Summary:

Use Synthetic Division to: ① Decide if a binomial is a factor (ex 3)

② Find the remainder (ex 4)

③ Find the quotient (ex 5)

④ Find remaining roots, given 1 root (ex 8)