

Section 2.3: Polynomial Equations

Essential Question:

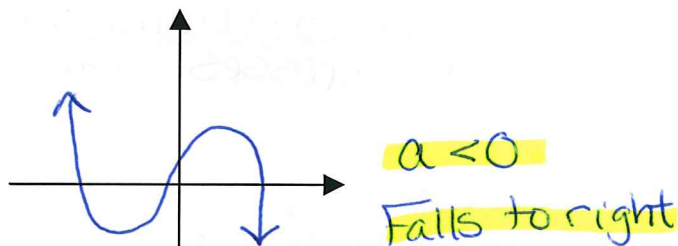
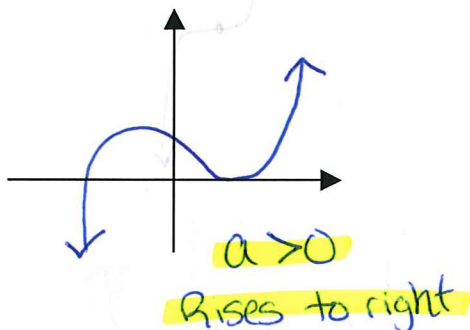
How do you sketch the graph of a cubic, quartic, or quintic function?

CUBIC

* $f(x) = ax^3 + bx^2 + cx + d$

* Shape is a sideways snake or "S" shape

* Unbroken curve



Example 1

Sketch a graph of $f(x) = x(x - 2)(x + 3)$

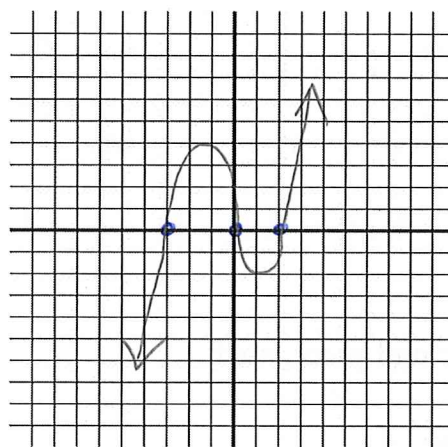
Zeros $x=0$ $x=2$ $x=-3$

Sign Analysis

$f(1) = (1)(1-2)(1+3)$
 $(+) (-) (+) = \text{Neg} \downarrow$

plug in one x -value between zeros

$f(-1) = (-1)(-1-2)(-1+3)$
 $(-) (-) (+) = \text{pos} \uparrow$

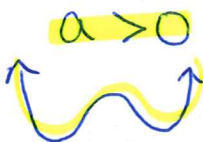


QUARTIC

* $f(x) = ax^4 + bx^3 + cx^2 + dx + e$

* Shape of graph is a "W" or a "M"

If...



Example 2

Graph $f(x) = (x + 3)(x + 1)(x - 1)(x - 2)$

x intercepts = -3 -1 +1 +2

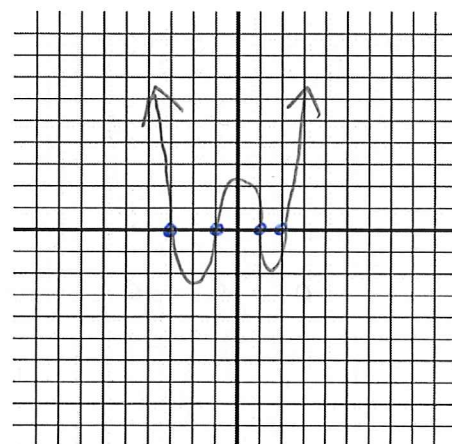
Since $x \cdot x \cdot x \cdot x = +x^4$

or sign analysis

$f(-2) = (+)(-)(-)(-) = \text{Neg} \downarrow$

$f(0) = (+)(+)(-)(-) = \text{Pos} \uparrow$

$f(1.5) = (+)(+)(+)(-) = \text{NEG} \downarrow$

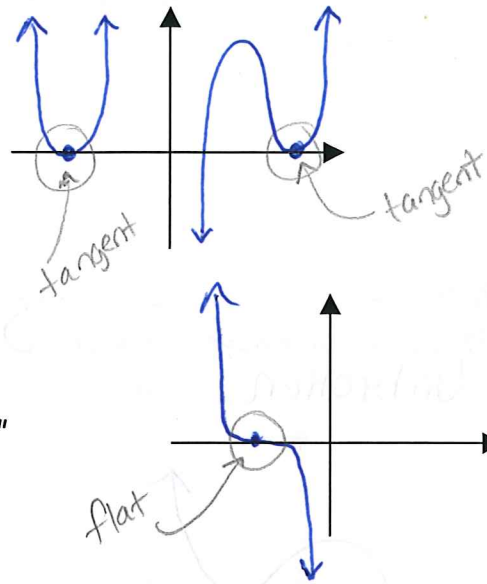


* Rough Sketch

Effect of a Squared or Cubed Term...

* If $P(x)$ has a squared term $(x - c)^2$
then $x = c$ is a DOUBLE root

-Graph is tangent to the x-axis



* If $P(x)$ has a cubed term $(x - c)^3$
then $x = c$ is a TRIPLE root

-Graph flattens out around "c"
and crosses x-axis

Example 3

Graph $f(x) = (x - 1)^2(x + 4)$

$x = 1$ $x = -4$

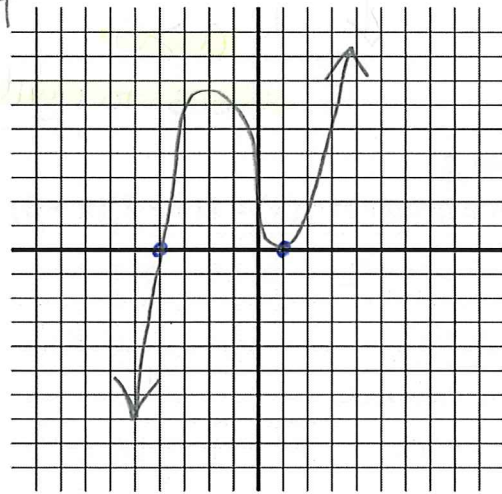
Double root → TANGENT

* Anything squared = pos

$f(2) = (+)(+) = \text{pos} \uparrow$

$f(0) = (+)(-) = \text{pos} \uparrow$

$f(-5) = (+)(-) = \text{neg} \downarrow$



Factor and graph each...

Example 4

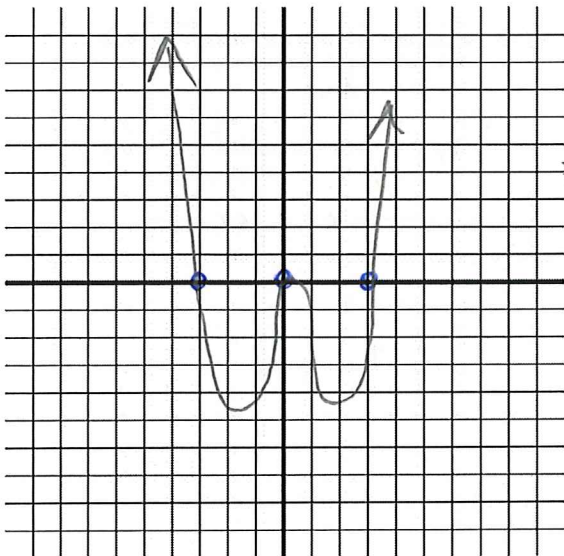
$f(x) = x^4 - 9x^2 = x^2(x^2 - 9) = x^2(x-3)(x+3)$

$a > 0$



$x = 0$ $x = \pm 3$

Double root → tangent



* Try graph without sign analysis by interpreting a-value

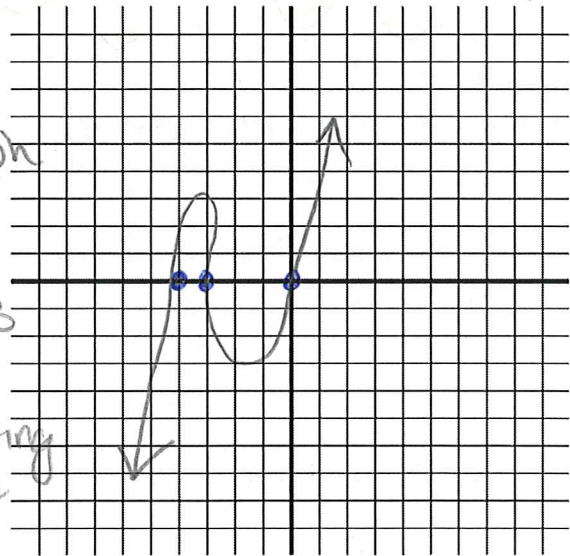
Example 5

$f(x) = x^3 + 7x^2 + 12x = x(x^2 + 7x + 12)$

$a > 0$



$x(x+4)(x+3)$
 $x = 0, -4, -3$



Quadratic

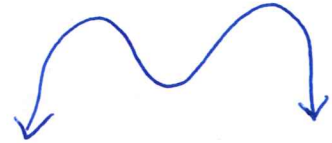
Cubic

Quartic

$a > 0$



$a < 0$



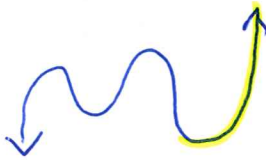
QUINTIC

$f(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$

If...

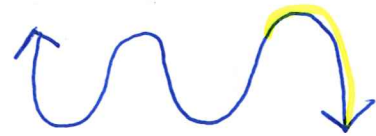
$a > 0$

rise
right



$a < 0$

fall
right



Example 6

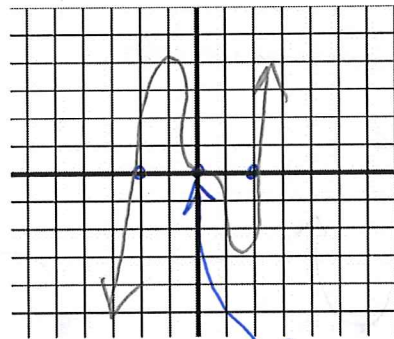
Graph $f(x) = x^5 - 4x^3$

$x^3(x^2 - 4) = x^3(x-2)(x+2)$

Triple root
= flat @ 0
 $x = 0, \pm 2$

$f(1) = (+)(-)(+) = \text{Neg} \downarrow$

$f(-1) = (-)(-)(+) = \text{Pos} \uparrow$



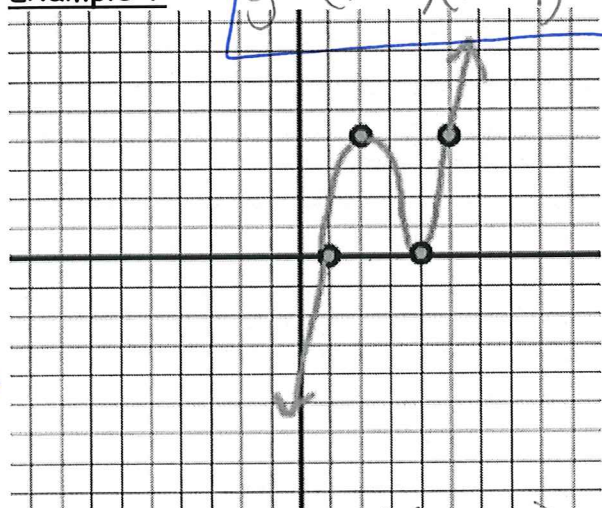
flat @ $x = 0$

Directions: For the following examples name the type of equation and write an equation for the given graph.

Example 7

$y = (x-1)(x-4)^2$

up
right
 $a = \text{pos}$



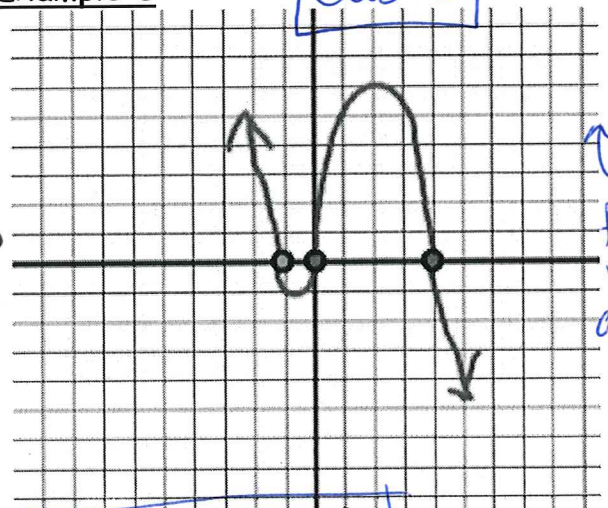
$x = 1$ $x = 4$ (double)

CUBIC

Example 8

CUBIC

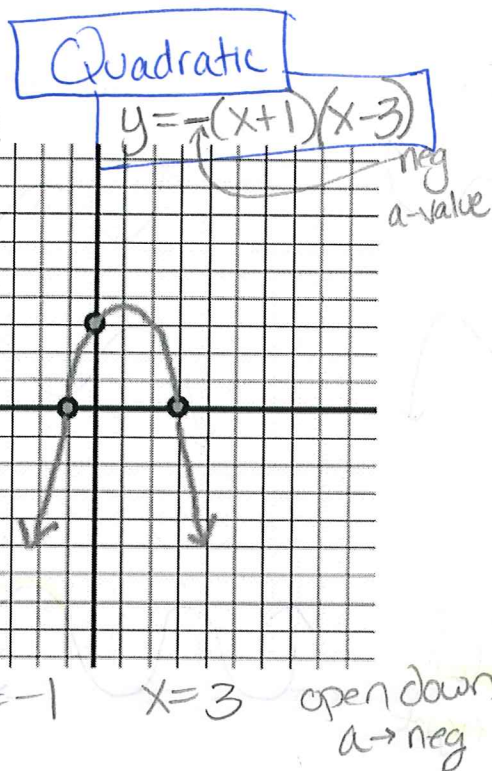
$x = -1$
 $x = 0$
 $x = 4$



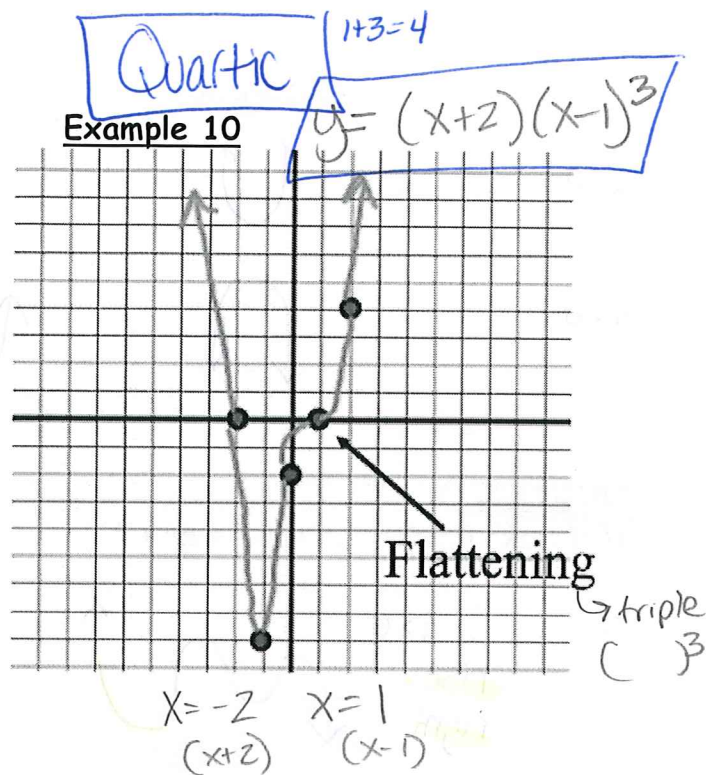
fall
right
 $a = \text{neg}$

$y = -x(x+1)(x-4)$
a-value negative

Example 9



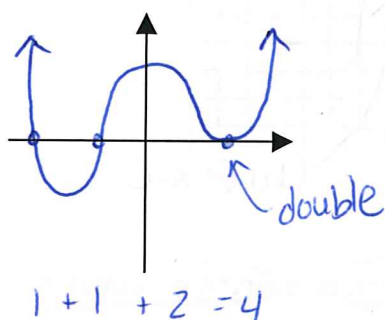
Example 10



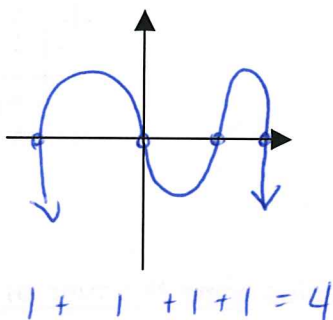
Example 11

Sketch a quartic equation with...

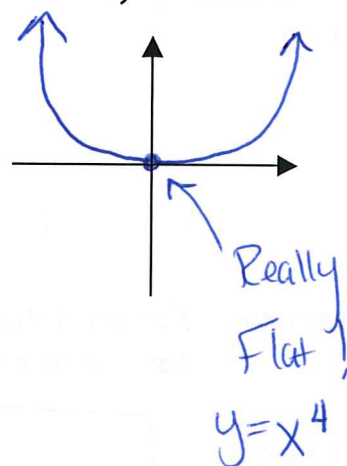
a) 3 zeros



b) 4 zeros



c) 1 zero



Section 2.3 Summary:

	cubic	Quartic	Quintic
$a > 0$			
$a < 0$			

- * Watch for double roots $()^2 \rightarrow$ tangent on x-axis
- * Watch for triple roots $()^3 \rightarrow$ flatten around x-axis

- 1) Factor to find x-intercepts (zeros by equation = 0)
- 2) Plot x-int then pick x-value between zeros for sign analysis
- 3) Sketch graph up if pos, down if neg.