Section 2.6: Solve by Factoring

Essential Question: How do you solve a function that is not factorable?

1) Group

$x^3 + 4x^2 - 9x - 36 = 0$

$x^2(x+4) - 9(x+4)$ *Notice Same factor in ( )

$(x^2-9)(x+4)$

Diff. of squares → $(x-3)(x+3)(x+4)$

$x=3$ $x=-3$ $x=-4$

Cubic equation ⇒ up to 3 roots

2) GCF → Always Check for GCF 1st

$x^3 + 2x^2 - 3x = 0$

$x(x^2+2x-3)=0$

$x(x+3)(x-1)=0$

$x=0$ $x=-3$ $x=1$

a·c = -3; 3 · -1; 3 + -1 = 2 = b

a=1 so factors right away

Review...

Factor $x^2 - 3x - 4 = 0$

-4 / +1 → $(x-4)(x+1)=0$

$x=4$ $x=-1$

Quadratic = 2 roots

3) Rewrite in Quadratic Form

a) $x^4 - 3x^2 - 4 = 0$

Quartic → $x^2 \cdot x^2$

-4 / +1 → $(x^2-4)(x^2+1)$

↳ D.O.S

$(x-2)(x+2)(x^2+1)$

$x=2$ $x=-2$

2 real roots

Quartic = 4 roots

2 imaginary roots → $x=\pm i$

$x^2+1=0$

$x^2=-1$

$x=\pm\sqrt{-1}$

b) $2x^4 - x^2 - 3 = 0$

New x^2 terms: a·c = -6; -3 · +2; -3 + 2 = -1 = b

$a \neq 1$ → group

$2x^4+2x^2-3x^2-3=0$

$2x^2(x^2+1)-3(x^2+1)=0$

$(x^2+1)(2x^2-3)=0$

$x^2+1=0$

$x^2=-1$

$x=\pm\sqrt{-1}=\pm i$

$2x^2-3=0$

$2x^2=3$

$x^2=3/2$

$x=\pm\sqrt{3}\,(\sqrt{2})$

$\sqrt{2}\,(\sqrt{2})$

$x=\pm\frac{\sqrt{6}}{2}$

Rational Root Theorem

- Finds rational numbers that are solutions of polynomial equations
- p/q is a solution where:
  - *p is a factor of the constant
  - *q is a factor of the leading coefficient

4) Solve $3x^3 + 8x^2 - 9x + 2 = 0$ (q, p)

p: ± 1 ± 2 (factors of 2)

q: ± 1 ± 3 (factors of 3)

$\frac{p}{q} = \pm\frac{1}{1}$ $\pm\frac{1}{3}$ $\pm\frac{2}{1}$ $\pm\frac{2}{3}$

± 1 ± 2

Synthetic Division (coefficients of the polynomial)

	3	8	−9	2
1	3	11	2	$4 \neq 0$
−1	3	5	−14	$16 \neq 0$
$\frac{1}{3}$	3	9	−6	0 = Remainder

*once you find a root stop

Quotient = $3x^2 + 9x - 6$

factor → $3(x^2 + 3x - 2)$ GCF

$a = 1$, $b = 3$, $c = -2$

$\frac{-3 \pm \sqrt{9 - 4(1)(-2)}}{2(1)} = \frac{-3 \pm \sqrt{17}}{2}$

$x = \frac{1}{3}$ one root

$x = \frac{-3 \pm \sqrt{17}}{2}$ + 2 roots

total = 3 roots (cubic)

Section 2.6 Summary:

The rational root theorem is a process to use to solve for the roots (x-intercepts) of polynomial functions. List all the possible roots by dividing the factors of the constant by the factors of the leading coefficients. Once you created the list of possible roots use synthetic division until you find a root (when remainder = 0). Finally, try to factor the quotient to find the remaining roots (or maybe use quadratic formula).