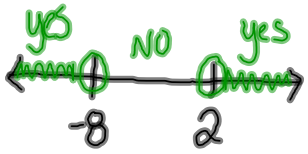
Solving Polynomial Inequalities
Section 3.2 Notes

Example 1
Solve $x^2 + 6x - 16 > 0$ ← open circle

1st: Find zeros of the polynomial, set P(x) = 0

$x^2+6x-16=0$
$(x+8)(x-2)$
$x=-8$
$x=2$



2nd: Test each interval

$f(-9) = 11 > 0$ True → shade
$f(0) = -16 > 0$ false
$f(3) = 11 > 0$ True → shade

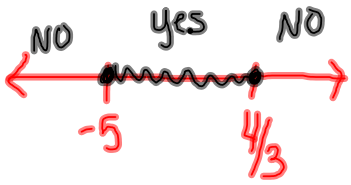
3rd: Write the solution as an inequality

$x < -8$ OR $x > 2$

*OR statement because 2 pieces

Example 2
$(4 - 3x)(5 + x) \geq 0$ ← closed •

$4-3x=0$ $5+x=0$
$x=\frac{4}{3}$ $x=-5$



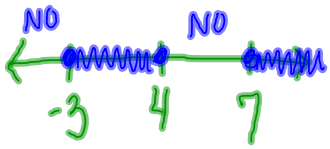
$f(-6) = -22 \geq 0$ False
$f(0) = 20 \geq 0$ True
$f(2) = -14 \geq 0$ False

$-5 \leq x \leq 4/3$

Example 3

$(x - 7)(x + 3)(x - 4) \geq 0$

$x = 7, -3, 4$ ↳ closed



$-3 \leq x \leq 4$ OR $x \geq 7$

$f(-4) = -88 \geq 0$ False

$f(0) = (-7)(3)(-4) = \text{positive} \geq 0$ True ↳shade

$f(5) = (-2)(8)(1) = \text{negative} \geq 0$ False

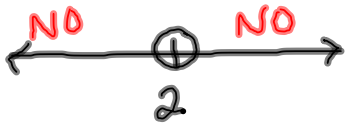
$f(8) = (1)(11)(4) \geq 0$ True ↳shade

Example 4

$x^2 - 4x + 4 < 0$

$(x-2)(x-2) = 0$

$x = 2 \quad x = 2$



* No shading & 2 has an open circle ∴

$f(0) = (0-2)(0-2) = 4 < 0$ False

$f(3) = (3-2)(3-2) = 1 < 0$ False

$\varnothing$ No solution

Example 5

$x^4 - 4x^2 \geq 0$ → closed circles

GCF = x^2

$x^2(x^2-4)=0$

$x^2(x-2)(x+2)=0$

$x=0 \quad x=2 \quad x=-2$

$x \leq -2$ OR $x \geq 2$

$x=0$

$f(-3) = (-3)^2(-3-2)(-3+2)$
$+ \quad - \quad - = + \geq 0$ True

$f(-1) = (-1)^2(-1-2)(-1+2)$
$+ \quad - \quad + = - \geq 0$ False

$f(1) = (1)^2(1-2)(1+2)$
$+ \quad - \quad + = - \geq 0$ False

$f(3) = 3^2(3-2)(3+2)$
$+ \quad + \quad + = + \geq 0$ True

Example 6

$$\frac{(x-2)(x-7)^2}{x-4} \geq 0$$ ← closed 0

1) Denominator $\neq 0$ (always open circle)

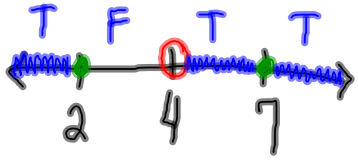
$x-4=0$ if $x=4$ so open circle on 4 because the denominator $\neq 0$

2) Find when the numerator = 0

$x-2=0 \quad x-7=0$

$x=2 \quad x=7$ * closed because original $\geq$

3) Plot ALL zeros for numerator and denominator



$x \leq 2$ OR $x > 4$

$f(0)$ $\frac{-+}{-} \geq 0$ True

$f(3)$ $\frac{++}{-} \geq 0$ False

$f(5)$ $\frac{++}{+} \geq 0$ True

$f(8)$ $\frac{++}{+} \geq 0$ True

Example 7

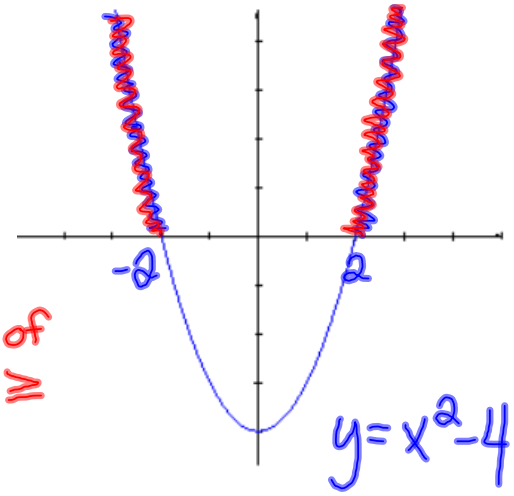
Use the graph to solve the inequality.

$x^2 - 4 \geq 0$

≥ 0 above the x-axis

include 2 and -2 because of original $\geq$

$x \leq -2$ OR $x \geq 2$



Example 8

Graph to solve the inequality.

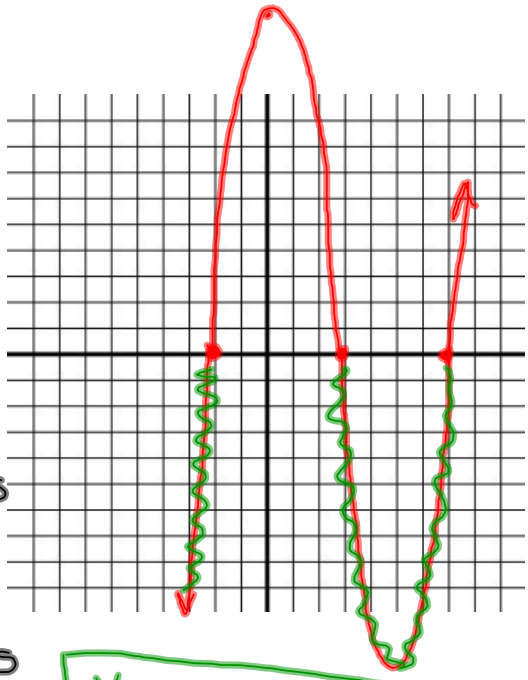
$(x - 3)(x + 2)(x - 7) < 0$

$x = 3, -2, 7$ → below x-axis

Sign Analysis

$f(0) = (0-3)(0+2)(0-7) = 42$ above x-axis

$f(4) = (4-3)(4+2)(4-7) = -18$ below x-axis



$x < -2$ OR $3 < x < 7$

HOMEWORK

P.103 #1-15 ODD
#16, 25-29,
31, 32 (GRAPH)