

Section 2.1: Polynomial Functions

Essential Question: ① What is a polynomial?
② How do you name a polynomial?
③ How can you evaluate a polynomial given a real#?

Polynomials

- Have nonnegative exponents
- Variables ONLY in Numerator
- General Form $a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x^1 + a_0$

Example 1

$$27x^5 + 10x^4 - 6x^3 + 2x^2 - x + 7.5$$

Terms:

$$27x^5, 10x^4, -6x^3, 2x^2, -x, 7.5$$

(6 terms)

Coefficients: 27, 10, -6, 2, -1, 7.5
(the # in front)

- Always write in Descending order
- Leading Term - Term with highest power
- Leading coefficient - coefficient of the leading term
- Degree of a polynomial - power of the leading term

<u>DEGREE</u>	<u>NAME</u>	<u>EXAMPLE</u>
0	<u>constant</u>	7
1	<u>Linear</u>	7x
2	<u>Quadratic</u>	$4x^2 + 7$
3	<u>Cubic</u>	$x^3 - 2x + 5$
4	<u>Quartic</u>	$5x^4 - 3x^3 + x^2$
5	<u>Quintic</u>	$8x^5 - 10$

* highest power = 5, but does not have to have an x^4, x^3, x^2, x or constant

Example 2

Name the term of the polynomial $7x^4 + 10x^3 - 6x^2 + 5x + 11$

Constant: 11

Linear Term: 5x

Quadratic Term: $-6x^2$

Cubic Term: $10x^3$

Quartic Term: $7x^4$

Reminder: Zeros of a function are where $P(x) = 0$ (set equation = 0)

Example 3

Decide if the function is a polynomial function and find the zeros.

A) $f(x) = 7x - 5$

yes

$$7x - 5 = 0$$

$$x = 5/7$$

B) $g(m) = m^3 - 25m$

yes

$$m^3 - 25m = 0$$

$$m(m^2 - 25) = 0$$

$$m(m+5)(m-5) = 0$$

$$m = 0, \pm 5$$

C) $g(h) = h - \frac{1}{h}$

NO

variable in denom.

$$h(0 = h - \frac{1}{h})$$

$$0 = h^2 - 1$$

$$0 = (h-1)(h+1)$$

$$h = \pm 1$$

Example 4

Given $f(x) = \frac{x^2 - 7x + 12}{x + 5}$

a) Where is the function undefined?

Set denominator = 0

$$x + 5 \neq 0 \quad x \neq -5$$

b) Find the zeros of the function.

Set numerator = 0

$$x^2 - 7x + 12 = 0$$

$$(x-4)(x-3) = 0$$

$$x = 4 \quad x = 3$$

Example 5

Find the values of function $f(x) = 2x^2 + 15$.

a) $f(3) = 33$

$$2(3)^2 + 15$$

$$2(9) + 15$$

$$18 + 15$$

$$33$$

b) $f(3i) = -3$

$$2(3i)^2 + 15$$

$$2(9i^2) + 15$$

$$18 \cdot (-1) + 15$$

$$-18 + 15$$

$$-3$$

c) $f(3n) = 18n^2 + 15$

$$2(3n)^2 + 15$$

$$2(9n^2) + 15$$

$$18n^2 + 15$$

d) $f(n+3) = 2n^2 + 12n + 33$

$$2(n+3)^2 + 15$$

$$2(n+3)(n+3) + 15$$

$$2(n^2 + 6n + 9) + 15$$

$$2n^2 + 12n + 18 + 15$$

$$2n^2 + 12n + 33$$

Example 6

$P(x) = 2x^4 - 3x^3 + x^2 + 6x - 6$

Find $P(2) = 2(2)^4 - 3(2)^3 + (2)^2 + 6(2) - 6 = 18$

Another way to find $P(x)$ is to use Synthetic Substitution

NOTE: You may only use synthetic substitution if x is a real number, x cannot be imaginary or a variable

$$\begin{array}{r|rrrrrr} 2 & 2 & -3 & 1 & 6 & -6 \\ & \downarrow & 4 & 2 & 6 & 24 \\ \hline & 2 & 1 & 3 & 12 & 18 \end{array} \leftarrow P(2)!$$

LAST EXAMPLE ☹

Use synthetic substitution to find $P(5)$ given: $f(x) = 6x^3 + 7x - 9$

$$\begin{array}{r|rrrr} 5 & 6 & 0 & 7 & -9 \\ & \downarrow & 30 & 150 & 785 \\ \hline & 6 & 30 & 157 & 776 = P(5) \end{array}$$

Section 2.1 Summary:

① A polynomial has variables only in the numerator, thus no negative exponents.

A polynomial should be written in descending order.

② The name of a polynomial is based on the degree (highest power).

$$\text{constant} = x^0$$

$$\text{Linear} = x^1$$

$$\text{Quadratic} = x^2$$

$$\text{Cubic} = x^3$$

$$\text{Quartic} = x^4$$

$$\text{Quintic} = x^5$$

③ To evaluate a polynomial, given a real number you can

A) substitute the value in for x and solve

OR B) use synthetic substitution (look @ example 6)