KEY

Section 3.2: Solving Polynomials of Degree 2 or higher

Essential Question:
How do you solve an inequality of a degree two or higher?

If the inequality is linear you just need to isolate the x-variable, but if you have an x^2 or higher you ***MUST FIND THE ZEROS!***

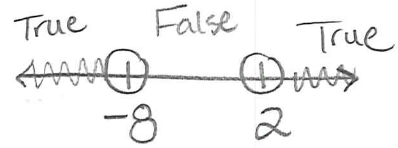
Example 1

Solve $x^2 + 6x - 16 > 0$ open

1st: Find zeros of the polynomial, set P(x) = 0

$x^2+6x-16=0$ $\quad x+8=0 \quad x-2=0$

$(x+8)(x-2) \quad x=-8 \quad x=2$



2nd: Test each interval

$f(-9)=(-9+8)(-9-2)$

$(-)(-)=\text{pos}>0$ True → shade

$f(0)=(+)(-)=\text{Neg}>0$ False → No shade

$f(3)=(+)(+)=\text{pos}>0$ True

$\boxed{x<-8 \text{ OR } x>2}$

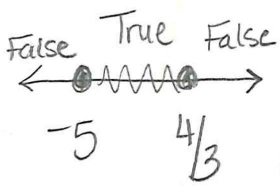
3rd: Write the solution as an inequality

Example 2

$(4-3x)(5+x) \geq 0$ closed

$4-3x=0 \quad 5+x=0$

$x=\frac{4}{3} \quad x=-5$



$f(-6)=(+)(-)=\text{NEG}\geq 0$ False

$f(0)=(+)(+)=\text{POS}\geq 0$ True

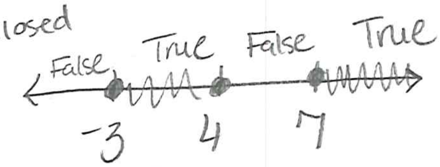
$f(2)=(-)(+)=\text{NEG}\geq 0$ False

$\boxed{-5\leq x\leq 4/3}$

Example 3

$(x-7)(x+3)(x-4) \geq 0$ closed

$x=7,-3,4$



$f(-4)=(-)(-)(-)=\text{NEG}\geq 0$ False → No shade

$f(0)=(-)(+)(-)=\text{POS}\geq 0$ True → shade

$f(5)=(-)(+)(+)=\text{NEG}\geq 0$ False → No shade

$\boxed{-3\leq x\leq 4 \text{ OR } x\geq 7}$

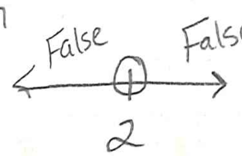
Example 4

$x^2 - 4x + 4 < 0$

$(x-2)(x-2)$ open

$(x-2)^2<0$

$x=2$



$(\text{pos})^2=\text{pos}<0$ False

$(\text{Neg})^2=\text{pos}<0$ False

No Shading

NO Answer → $\boxed{\text{No Solution } \emptyset}$

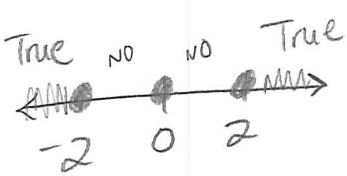
Example 5

$x^4 - 4x^2 \geq 0$

$x^2(x^2-4)$ closed

$(x^2)(x-2)(x+2)\geq 0$

$x=0, 2, -2$



$f(-3)=(+)(-)(-)=\text{POS}\geq 0$ True

$f(-1)=\text{NEG}\geq 0$ False

$f(1)=\text{NEG}\geq 0$ False

$f(3)=\text{POS}\geq 0$ True

$\boxed{x\leq -2 \quad x=0 \quad x\geq 2}$

Example 6

$$\frac{(x-2)(x-7)^2}{x-4} \geq 0$$

Closed

1) Denominator $\neq 0$ (always open circle)

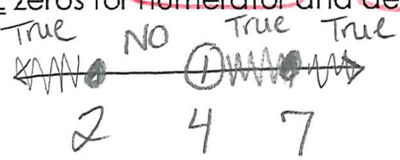
$x-4=0 \quad x=4$

2) Find when numerator = 0

$(x-2)(x-7)^2=0 \qquad x=2$

$x-2=0 \quad x-7=0 \qquad x=7$

3) Plot ALL zeros for numerator and denominator



$\boxed{x \leq 2 \text{ OR } x > 4}$

Test intervals

$f(1) = \frac{(-)(+)}{(-)} = \frac{NEG}{NEG} = pos \geq 0$ True

$f(3) = \frac{(+)(+)}{(-)} = NEG \geq 0$ False

$f(5) = \frac{(+)(+)}{(+)} = POS \geq 0$ True

$f(8) = \frac{(+)(+)}{(+)} = POS \geq 0$ True

Example 7

Use the graph to solve the inequality.

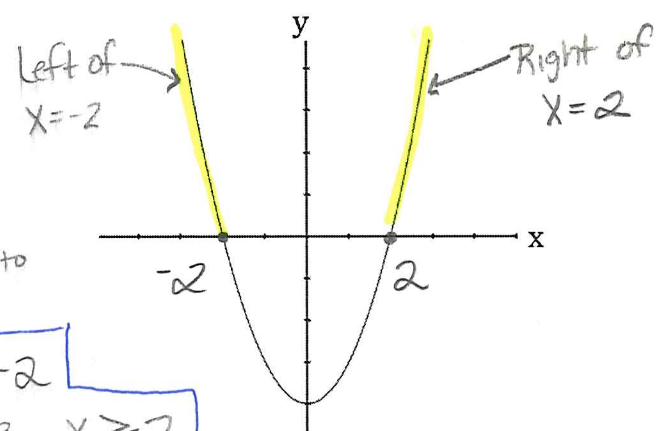
$x^2 - 4 \geq 0$

equal to — include roots — = to — equal to

* Above x-axis

* if $\leq 0 \rightarrow$ Below x-axis

$\boxed{x \leq -2 \text{ OR } x \geq 2}$



Example 8

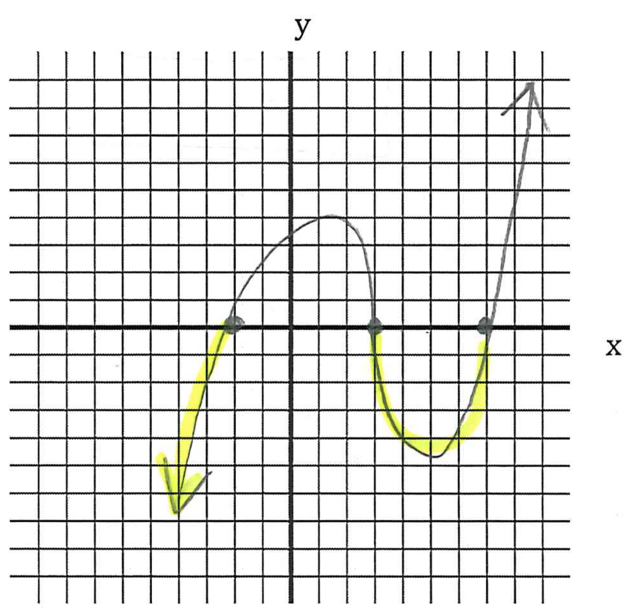
$(x-3)(x+2)(x-7) \boxed{< 0}$ Below x-axis

$x = 3, -2, 7$

$x \cdot x \cdot x = +x^3 \quad a \rightarrow pos$

left of $x=-2$ Between $x=3$ & 7

$\boxed{x < -2 \text{ OR } 3 < x < 7}$



Section 3.2 Summary:

To graph an inequality of degree 2 or higher...

① First factor to find the roots and plot them on a number line. ② If $>$ or $<$ open circle, if $\leq$ or $\geq$ Closed circle. ③ Then test the intervals to know where to shade (if "true" Shade, if "false" no shade).

④ Finally, write an inequality that represents the shaded region(s).