

Section 4.1: Functions

Essential Question:

What is a function?

- It is a relation that assigns each x-value (Domain) with _____
y-value (Range)
- $f(x)$ is the value of the function of x

Remember: $f(x)$ represents y ... $f(x) = y$

- "f" is the _____ of the function

RELATION: is a correspondence or a set of ordered pairs (x, y)

$\{(2,1), (3,4), (6,4), (7,7)\}$

This relation is a function because

$\{(1,3), (5,2), (5,7), (7,5)\}$

This relation is NOT a function because

NOTATION

- Function f is a set of ordered pairs (x,y) where x is an element of the _____
and y is the corresponding element in the _____
- Written as $y = f(x)$

Using different VARIABLES

- $v = g(u)$ means the function is a set of ordered pairs _____
- Name of function is _____
- u -values are the _____ and v -values are the _____

Examples

Determine the domain of the function.

1) $h(x) = \frac{5}{x-9}$

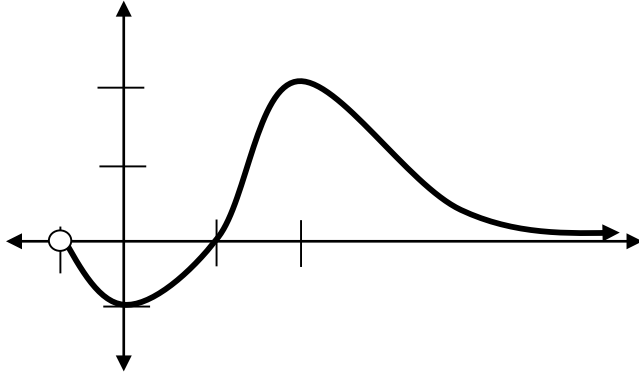
2) $m(t) = \sqrt{2t-3}$

3) $h(x) = \sqrt{x^2-4}$

Examples #4 - 6

Use the graph to determine the domain, range, and zeros for #4 - 6.

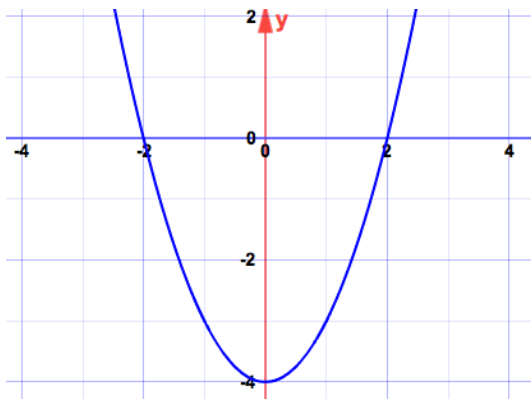
4)



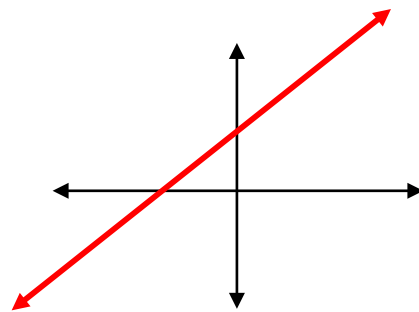
Domain: find by projecting...

Range: find by projecting...

5)



6)



Examples #7 - 9

Are the following relations described by an equation a function?

7) $y = 3x + 1$

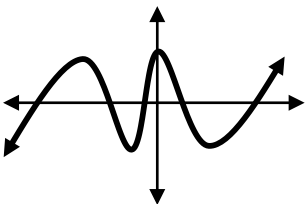
8) $y = |x| + 2$

9) $x = |y| - 3$

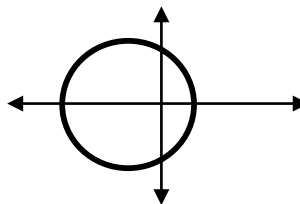
Example 10

Are the following graphs functions?

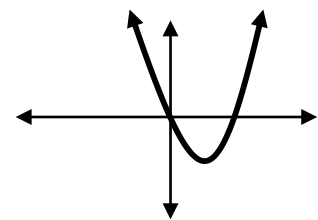
a)



b)



c)



Section 4.1 Summary:

Section 4.2: Operations on Functions

Essential Question:

Given f and g are functions...

1. **Sum** of f and g : $(f + g)(x) = \underline{\hspace{2cm}}$

2. **Difference** of f and g : $(f - g)(x) = \underline{\hspace{2cm}}$

3. **Product** of f and g : $(f \cdot g)(x) = f(x) \cdot g(x)$

4. **Quotient** of f and g : $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, provided $g(x) \neq 0$

Example: Given $f(x) = x^2$ and $g(x) = x + 1$

Find the sum
 $(f + g)(x) =$

Find the difference
 $(f - g)(x) =$

Find the product
 $(fg)(x) =$

The Composite Function

- Given functions f and g , denoted $(f \circ g)(x) = f(g(x))$
- Read as "f of g" (the open circle is **NOT** a multiplication symbol)
- x is in the domain of function g and $g(x)$ is in the domain of function f

Example: use the same functions f and g above

$$(f \circ g)(x) =$$

Homework (in class) p127 CE #5 - 10 all

Given $f(x) = x^2 + x$ and $g(x) = x + 1$

5) $(f + g)(x) =$

9) a) $f(g(2)) =$

6) $(f - g)(x) =$

b) $(f \circ g)(x) =$

7) $(f \cdot g)(x) =$

10) a) $(g \circ f)(2) =$

8) $(f \div g)(x) =$

b) $(g \circ f)(x) =$

Section 4.2 Summary:

Section 4.3: Reflecting Graphs and Symmetry

Essential Question:

1) Reflection over the x-axis

- $y = -f(x)$ is a reflection of $y = f(x)$ over the _____
- (____ , ____) is the reflected coordinate of (x, y)

Examples

$$y = x^2 - 4$$

OR

$$y = 2x + 1$$

2) Entire graph above the x-axis

-

Example

$$y = x^2 - 5 \quad \text{becomes} \quad \underline{\hspace{2cm}}$$

3) Reflection over the y-axis

- $y = f(-x)$ is a reflection of $y = f(x)$ over the _____
- (____ , ____) is the reflected coordinate of (x, y)

Examples

$$y = 2^x$$

OR

$$y = (x + 2)^2$$

4) Reflection over the line $y = x$

- Interchange the x and y
- (____ , ____) is the reflected coordinate of (x, y)

Examples

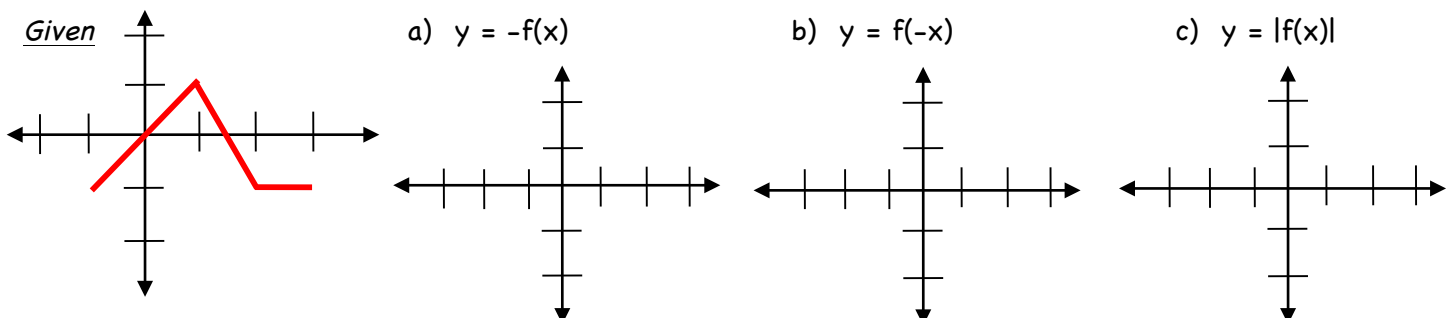
$$y = 3x + 1$$

OR

$$y = x^3$$

Example 1

Given the graph of $y = f(x)$ sketch the graph of...

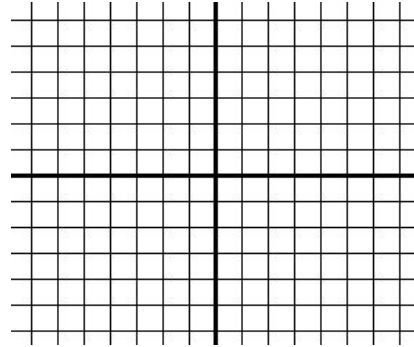
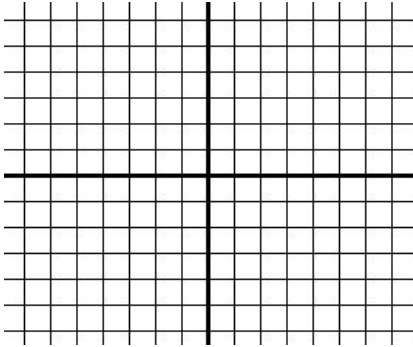


Example 2

Sketch the graph of the equation and the reflection of the graph over $y = x$.
Give the equation of the new graph.

a) $y = 2x - 1$

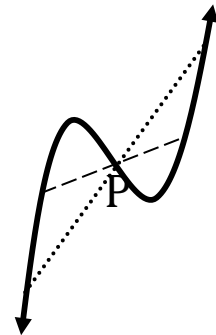
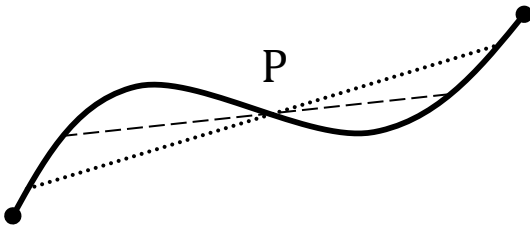
b) $y = x^2 - 4x$



Point of Symmetry (POS)

A point P is called POS of a graph if you can pair the points so P is the midpoint of the segment joining each pair of points.

Examples



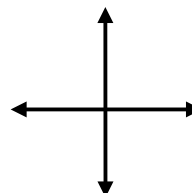
Axis of Symmetry (AOS) or Line of Symmetry (LOS)

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-

How to Test for Symmetry of a Graph

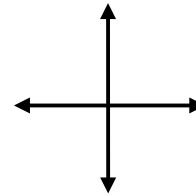
Symmetric to the x-axis

-
- Test: leave x alone and substitute _____
- Do you get an equivalent equation?



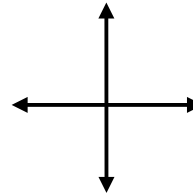
Symmetric to the y-axis

-
- Test: leave y alone and substitute _____
- Do you get an equivalent equation?



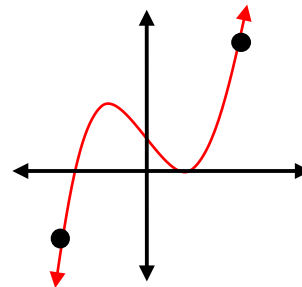
Symmetric to the line y = x

-
- Test: Interchange x and y
- Do you get an equivalent equation?



Symmetric to the origin

-
- Test: Substitute ___ for x and ___ for y
- Do you get an equivalent equation?



Examples #3 - 6

Test for symmetry with...

a) the x-axis

b) the y-axis

c) line $y = x$

d) the origin

#3

$$x^4 + y^4 = 1$$

#4

$$xy^3 = 1$$

#5

$$x(x + y) = 1$$

#6

$$y = x^2 + 7$$

Example 7

Find the AOS for

$$y = x^2 - 8x - 7$$

Example 8

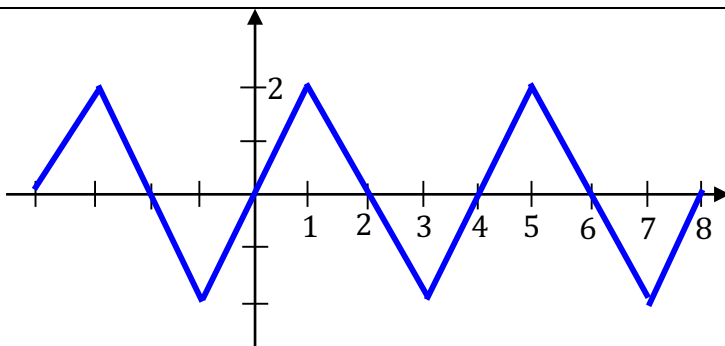
Find the POS for

$$y = x^3 - 6x^2 + 5x + 7$$

Section 4.3 Summary:

Section 4.4: Periodic Functions; Stretching & Translating Graphs

Essential Question:



This graph is _____ ; it has a positive period P

Fundamental Period is the _____ period of a function

Example 1

Find the fundamental period of $f(x)$.

Example 2

Find $f(103)$

Example 3

Find $f(106)$

AMPLITUDE is the average of the maximum and minimum values of a periodic function

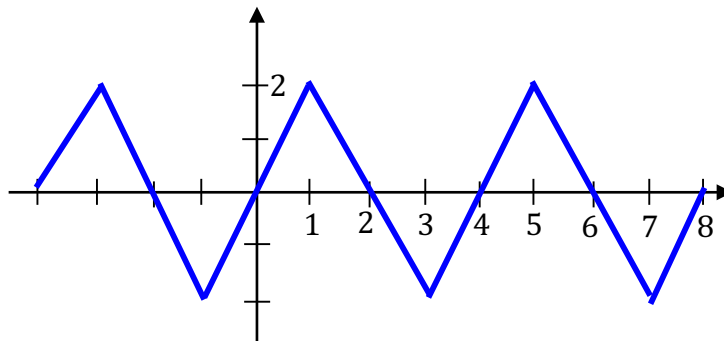
A =

M:

m:

Example 4

Find the amplitude of $f(x)$



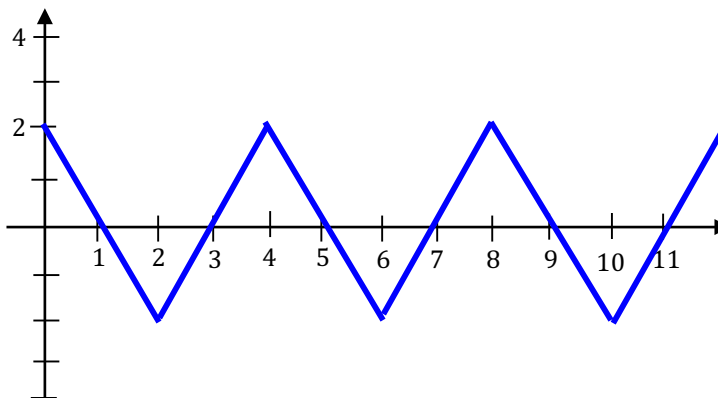
STRETCHING AND SHRINKING A GRAPH

Vertical stretch or shrink: $y =$ _____

P =

A =

$y = 2f(x)$



$y = \frac{1}{2} f(x)$

If _____ the graph is vertically _____

If _____ the graph is vertically _____

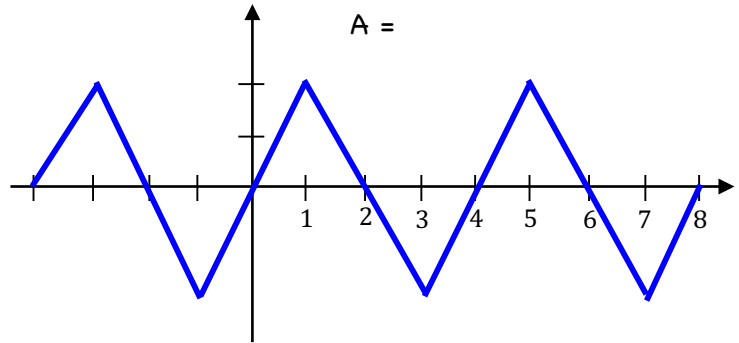
Note: _____

Horizontal stretch or shrink: $y =$ _____

$P =$

$y = f(2x)$

$A =$



$y = f(\frac{1}{2} x)$

If _____ the graph is horizontally _____

If _____ the graph is horizontally _____

Note: _____

Changing Period and Amplitude of a Periodic Function

If a periodic function f has a period P and amplitude A ...

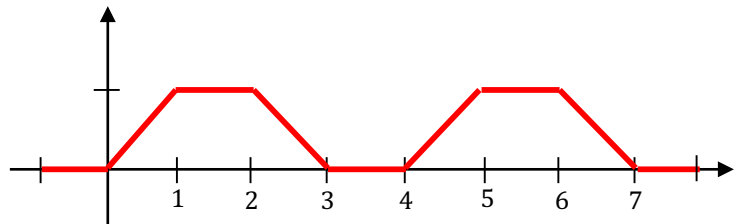
Then $y = cf(x)$ has period _____ and amplitude _____

Then $y = f(cx)$ has period _____ and amplitude _____

For examples #5 - 7 use the graph below:

Example 5

Find the period and amplitude of $f(x)$



Example 6

Find the period and amplitude of

$y = 2f(x)$

Example 7

Find the period and amplitude of

$y = f(\frac{1}{2} x)$

TRANSLATING A GRAPH



$y - k = f(x - h)$ is the same as:

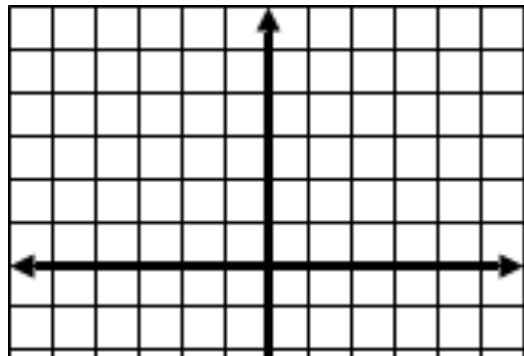
Example 8

Graph $y = |x|$

$y =$

$y =$

$y =$



Note: When doing multiple translations think in terms of _____ of _____

Reminder: $(x - h)^2 + (y - k)^2 = r^2$

A circle with center _____ and radius = _____

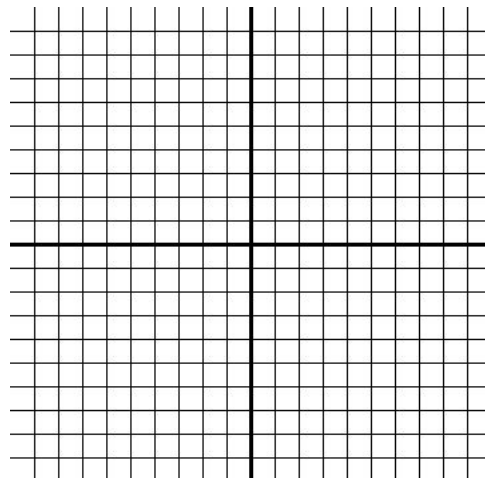
Section 4.4 Summary:

Section 4.5: Inverses and One-to-one Functions

Essential Question:

INVERSES

- Given a function $f(x)$ the inverse is _____
- Find the inverse by...
 -
 -
 -



Example 1

Find the inverse of $f(x) = 3x - 1$

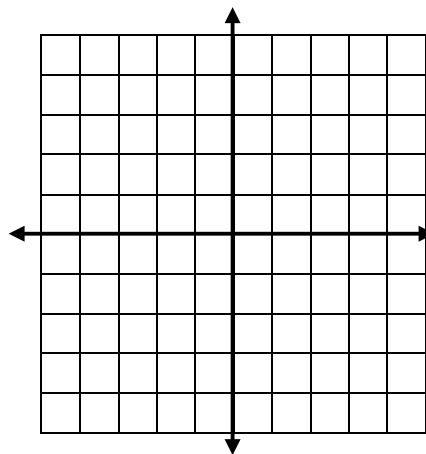
Graph $f(x)$ and $f^{-1}(x)$

Find $f(1) =$ $f^{-1}(2) =$

Example 2

Graph $f(x) = x^2 + 2$ and its reflection over the line $y = x$

Find the $f^{-1}(x)$



One - To - One Functions

A function that has an inverse.

(The given equation is a function and the inverse is a function.)

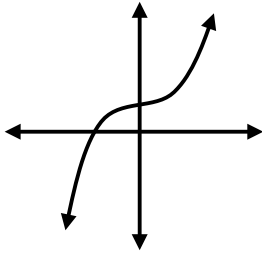
HOW TO CHECK IF A FUNCTION IS 1-to-1

$f(x)$ is a function if it passes the VLT (vertical) but...

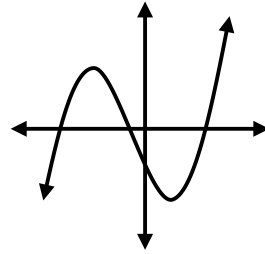
- $f^{-1}(x)$ is NOT a function if $f(x)$ _____
- $f^{-1}(x)$ is a function if $f(x)$ _____

Example 3

a)



b)



Example 4

Suppose $f(x)$ has an inverse.

If $f(0) = -1$ and $f(-1) = 2$ find...

a) $f^{-1}(-1)$

b) $f^{-1}(f(0))$

c) $f(f^{-1}(-1))$

Section 4.5 Summary: