## Section 5.2: Growth and Decay Functions

Essential Question: What are the growth and decay models? what is a rational exponent?

**Model for GROWTH**

$A(t) = A_0(1+r)^t$

**Model for DECAY**

$A(t) = A_0(1-r)^t$

$A_0$ = initial amount (start)

r = rate as a decimal

t = time (in years)

% to decimal → move 2 place left → .08

**Examples**

1. Jeans at a department store increase at a rate of 8% per year. The current price of jeans is $25. What will be the cost of jeans four years from now? $25(1+.08)^4 = \$34.01$

2. You bought a boat for $9800 three years ago and have decided to upgrade to a newer boat. To make your new purchase you put down the money you make from the sale of the old boat. The bank informed you that boats decrease about 20% per year. How much money do you expect to get from your boat sale?

$9800(1-.20)^3 = \$5017.60$

3. A train ticket costs $275 on average. The annual rate of increase is 18%. What will be the average rate in 10 years?

$275(1+.18)^{10} = \$1439.30$

4. Sam purchased a motorcycle for $7500 eight years ago and now plans to sell it. The annual rate of decrease for a motorcycle is approximately 21% depending on the condition. How much should Sam sell is motorcycle for?

$7500(1-.21)^8 = \$1137.83$

***RATIONAL EXPONENTS***

exponent → $c^{m/n} = \sqrt[n]{c^m}$ OR $(\sqrt[n]{c})^m$ ← index

**Examples**

5. $4^{\frac{1}{2}}$

$\sqrt[2]{4^1}$ OR $(\sqrt[2]{4})^1$

$= \sqrt[2]{4}$ $= \sqrt[2]{4}$

$= 2$ same $= 2$

6. $4^{-3/2} = (\sqrt[2]{4})^{-3} = 2^{-3}$

power

index → 2, radical → 4

$= \frac{1}{2^3} = \frac{1}{8}$

7. $-9^{1/2} = -1 \cdot 9^{1/2}$ (not same as $(-9)^{1/2}$)

$= -1 \cdot \sqrt{9}$

$= -1 \cdot 3$

$= \boxed{-3}$

8. $\left(3^{1/2} \cdot 5^{1/2}\right)^2 = \left(\sqrt{3} \cdot \sqrt{5}\right)^2$

$= \left(\sqrt{15}\right)^2 = \boxed{15}$

9. $\left(3^{1/2} + 5^{1/2}\right)^2 = \left(\sqrt{3} + \sqrt{5}\right)^2$

$= \left(\sqrt{3} + \sqrt{5}\right)\left(\sqrt{3} + \sqrt{5}\right)$ FOIL

$= 3 + \sqrt{15} + \sqrt{15} + 5 = \boxed{8 + 2\sqrt{15}}$

10. $\dfrac{x^{1/3}}{2x^{-2/3}} = \dfrac{x^{1/3} \cdot x^{2/3}}{2} = \dfrac{x^{3/3}}{2} = \boxed{\dfrac{x}{2}}$

**Solve**

11. $3^{2x} = 3^{12}$

Same base → exp. equal

$2x = 12$ $\boxed{x = 6}$

12. $9^x = 3^5$ (need same base)

$3^2 = 9$ (change base)

$\left(3^2\right)^x = 3^5$

$3^{2x} = 3^5 \rightarrow 2x = 5$

$\boxed{x = 5/2}$

13. $\left(x^{2/3}\right)^{3/2} = (9)^{3/2}$

Raise the exponent to its reciprocal

$x^1 = 9^{3/2}$

$x = \left(\sqrt{9}\right)^3 = (3)^3 = \boxed{27}$

14. $\left(x^{-1/2}\right)^{-2/1} = (4)^{-2/1}$

$x^1 = 4^{-2}$

$x = \dfrac{1}{4^2} = \boxed{\dfrac{1}{16}}$

**Section 5.2 Summary:**

Growth (increase) $= A_0(1+r)^t$

Decay (decrease) $= A_0(1-r)^t$

$A_0$ = initial amount

$r$ = rate as a decimal

$t$ = time in years

Rational Exponent (fraction) * to be simplified, no rational exponents

$b^{x/y} = \sqrt[y]{b^x}$ or $\left(\sqrt[y]{b}\right)^x$

depending on problem one might be easier to simplify than the other