Section 5.5: Logarithmic Functions

Essential Question: What is a logarithm?

What is the difference between common and natural log?

A log is an exponent

If $b^x = y$ (exponential form) then… $\log_b y = x$ (log form)

base (of the exponent) — base (of the log) — equals the exponent

Read as: "log base b of y is x"

Exp. Form	Log Form
$2^3 = 8$	$\log_2 8 = 3$
$3^4 = 81$	$\log_3 81 = 4$
$5^{-2} = 1/25$	$\log_5 (1/25) = -2$

***Common* Logarithms**
-Base 10
-Written as $\log x = y$ means $10^y = x$

Natural Logarithms
-Base e
-Written as $\ln x = y$ means $e^y = x$

Examples
Solve for x.

1) $\log 10 = x$
no base → 10
$10^x \stackrel{?}{=} 10$ $x = 1$

2) $\ln e^2 = x$
base 'e'
$e^x \stackrel{?}{=} e^2$
$x = 2$

3) $\ln e = x$
base 'e'
$e^x \stackrel{?}{=} e$
$x = 1$

Short Cut… $\log_a a^b =$ b

If the bases are the same, then the answer is the exponent.

Examples

4) $\log_2 16 = ? = x$
$2^x = 16$
$x = 4$

5) $\log 1000 = x$
base 10 if not written
$10^x = 1000$ $x = 3$

6) $\log_7 1 = x$
$7^x = 1$ $x = 0$

7) $\log_2 2^{11} = ? = x$
$2^x = 2^{11}$
$x = 11$

8) $\log_{49} 7 = ? = x$
$49^x = 7$
$(7^2)^x = 7$
$7^{2x} = 7^1$
$2x = 1$
$x = 1/2$

9) $\log .1 = ? = x$
$10^x = .1$
$10^x = \frac{1}{10}$
$10^x = 10^{-1}$
$x = -1$

Examples #10 – 12 use a calculator

natural log key "LN"

10) log 14 = ?= x 11) log 250 = 2.40 12) ln 4 = 1.39

$10^x = 14$ *need calc with log key

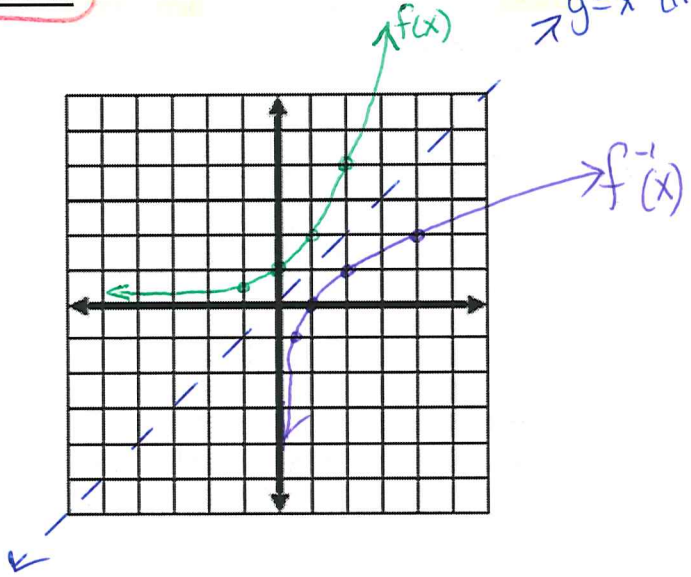
$x = 1.15$

Exponential and Logarithmic Equations are **INVERSES**

Inverses reflect over $y=x$ line

Ex 13

Graph $y = 2^x$ and its inverse.



$f(x) = 2^x$

x	y
-1	½
0	1
1	2
2	4

interchange x & y

$f^{-1}(x)$

x	y
½	-1
1	0
2	1
4	2

D: $\{x \mid \mathbb{R}\}$ D: $\{x \mid x>0\}$

R: $\{y \mid y>0\}$ R: $\{y \mid \mathbb{R}\}$

Asymptote: $y=0$ (x-axis) Asymptote: $x=0$ (y-axis)

Examples

Solve without a calculator. Leave answers in terms of e.

natural log = e base

14) log x = 2

base "10"

$10^2 = x$

$x = 100$

15) log |x| = 4

$10^4 = |x|$

$10000 = |x|$

$x = \pm 10{,}000$

16) ln x = 3

$e^3 = x$

may leave in exp. form

Solve using a calculator.

17) log x = 2.3

$10^{2.3} = x$

never leave answer w/ decimal exponent

$10^{2.3} \approx 199.53$

Section 5.5 Summary:

A logarithm is the inverse of an exponential function.

$b^x = y$ (exponential form) $\longrightarrow$ $\log_b(y) = x$ (log form)

Common log = base "10" Natural Log = base "e"

$\log(y)=x$ base does not have to be written $\ln(y)=x$ natural log