Section 5.6: Laws of Logarithms

Essential Question: What are the laws of logarithms?

Given M and N are positive real numbers, $b > 0$ and $b \neq 1$

1) $\log_b MN = \log_b M + \log_b N$

2) $\log_b (M/N) = \log_b M - \log_b N$

3) $\log_b M = \log_b N$ if $M = N$ (drop the logs)

4) $\log_b M^k = k \log_b M$
(power goes to the front)

Examples
Express in terms of *log*(M) and *log*(N).

1) $\log M^3$

$3 \log M$

2) $\log(3MN)$

$\log 3 + \log M + \log N$

3) $\log\sqrt{MN} = \log (MN)^{1/2}$

$\frac{1}{2}(\log M + \log N)$

$\frac{1}{2}\log M + \frac{1}{2}\log N$

Examples
Express as a single logarithm or a rational number.

4) $\log_5 2 + \log_5 7$

$\log_5(2 \cdot 7) = \log_5(14)$

5) $\ln e^5 = 5$ (short cut)
(base e)

OR $5 \cdot \ln e = 5 \cdot 1 = 5$

6) $\log_6 12 + \log_6 3$

$\log_6(12 \cdot 3) = \log_6 36 = 2$

$6^x \stackrel{?}{=} 36$

$x = 2$

7) $\log C - 3 \log W$

$\log C - \log W^3$

$\log\left(\frac{C}{W^3}\right)$

Examples
Express y in terms of x.

8) $\log y = 3 \log x$

$\log y = \log x^3$

$y = x^3$

9) $\ln y = \frac{1}{3}(\ln 7 + \ln x)$

$\ln y = \frac{1}{3}(\ln 7x)$

$\ln y = \ln (7x)^{1/3} \rightarrow y = (7x)^{1/3}$

$y = \sqrt[3]{7x}$

Solve for the value of x.

10) $\log_2 (x + 3) + \log_2 6 = 4$

$\log_2 6(x+3) = 4$

$\log_2 (6x+18) = 4$

$2^4 = 6x+18$

$16 = 6x+18$

$-2 = 6x$

$x = \frac{-2}{6} = \boxed{\frac{-1}{3}}$

11) $\log_2 x + \log_2 (x - 2) = 3$

multiply

$\log_2 (x^2-2x) = 3$

$2^3 = x^2-2x$

$0 = x^2-2x-8$

$0 = (x-4)(x+2)$

$\boxed{x=4}$ $x=-2$ extraneous

Check

$\log_2 4 + \log_2 (4-2)$

$\log_2 4 + \log_2 (2)$

okay, but

$\log_2 (-2) + \log_2 (4-2)$

cannot take log of neg #

Solve the following logarithms.

12) $\log_2 8 = x$

$2^x = 8$

$\boxed{x=3}$ → Change of base

$\frac{\log 8}{\log 2} = \boxed{3}$

(use calculator)

13) $\log_2 7$

$2^x = 7$

→ not an integer exponent

→ must change base and use calc

$\frac{\log 7}{\log 2} \approx \boxed{2.807}$

Section 5.6 Summary:

What are the laws of logarithms?

① $\log_b MN = \log_b M + \log_b N$ .... logs of the same base being added can be simplified with multiplication

② $\log_b \left(\frac{M}{N}\right) = \log_b M - \log_b N$ .... logs of the same base being subtracted can be simplified with division

③ If $\log_b M = \log_b N$, then $M=N$ .... If two log of the same base are equal, then the "of" parts are equal. (Drop the log)

④ $\log_b M^K = K \cdot \log_b M$ .... The coefficient of a log can be moved as the exponent of the log.