Key

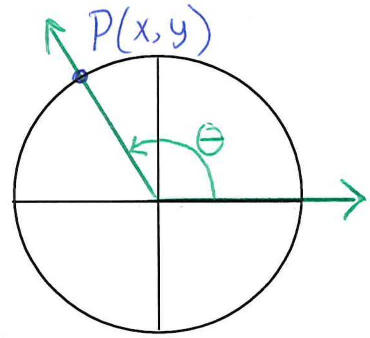
Section 7.3: The Sine and Cosine Functions

Essential Question: How do you find the $\sin\theta$ and $\cos\theta$ at a given point?

<u>The Sine and Cosine Functions</u>

- Used to describe repetitive patterns (i.e. sound waves)
- Sine (<u>sin</u>)
- Cosine (<u>cos</u>)

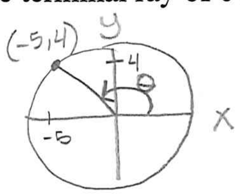
If P(x, y) is on a circle and θ is an angle in standard position, then



$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$

Remember an equation of circle is…
$x^2 + y^2 = r^2$

Example 1
If the terminal ray of θ in standard position passes through (-5, 4) find $\sin\theta$ and $\cos\theta$.



$x^2 + y^2 = r^2$
$(-5)^2 + (4)^2 = r^2$
$25 + 16$
$41 = r^2$
$r = \pm\sqrt{41}$

x y

choose $r = +\sqrt{41}$ (radius always positive)

$\sin\theta = \frac{y}{r} = \frac{4}{\sqrt{41}} \frac{(\sqrt{41})}{(\sqrt{41})}$ $\cos\theta = \frac{x}{r} = \frac{-5}{\sqrt{41}} \frac{(\sqrt{41})}{(\sqrt{41})}$

$\boxed{\sin\theta = \frac{4\sqrt{41}}{41}}$ $\boxed{\cos\theta = \frac{-5\sqrt{41}}{41}}$

Example 2
If θ is in quadrant III and $\cos\theta = \frac{-3}{5}$ find the $\sin\theta$.

x, r

(x, y)
−, −

$x^2 + y^2 = r^2$
$(-3)^2 + y^2 = 5^2$
$9 + y^2 = 25$
$-9 \qquad -9$

$y^2 = 16$
$y = \pm\sqrt{16}$
$y = -4$
(choose neg. Q3)

$\sin\theta = \frac{y}{r}$

$\boxed{\sin\theta = \frac{-4}{5}}$

Example 3
If θ is a 4th quadrant angle and $\sin\theta = -\frac{5}{13}$ find $\cos\theta$.

y, r

(x, y)
+ −

$y = -5$ $r = 13$

$\cos\theta = \frac{x}{r}$ ← need x-value

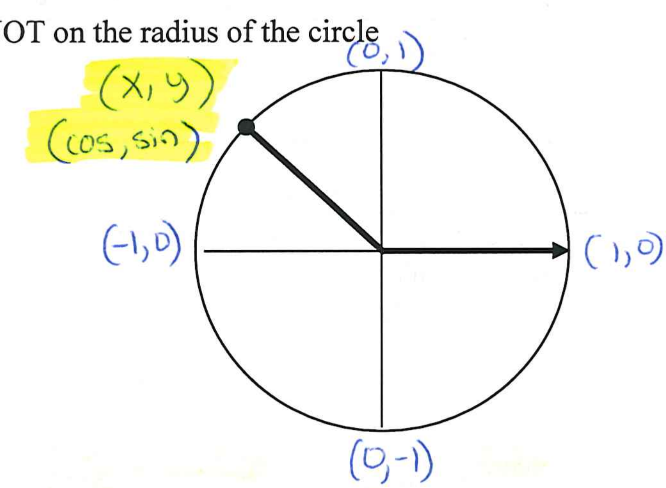
$x^2 + y^2 = r^2$
$x^2 + (-5)^2 = 13^2$
$x^2 + 25 = 169$
$-25 \qquad -25$

$x^2 = 144$
$x = \pm\sqrt{144}$

$x = +12$
(x positive in Q4)

$\boxed{\cos\theta = \frac{12}{13}}$

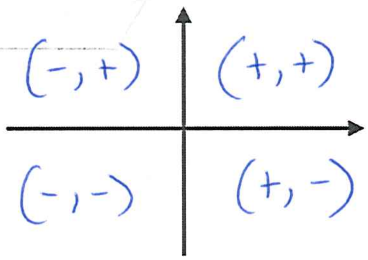
☞ The values of sinθ and cosθ depend only on θ NOT on the radius of the circle



UNIT CIRCLE

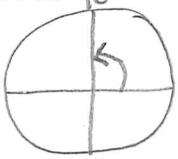
- Radius = 1
- Equation $x^2+y^2=1$
- $\sin\theta = y$ $\qquad$ $\cos\theta = x$
- Trig functions are called circular functions

Value of sinθ = y and cosθ = x



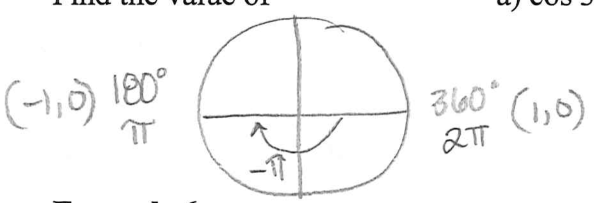
Example 4

Find the value of 90° (0,1) $\qquad$ a) cos 90° (x) $\boxed{0}$ $\qquad$ b) sin 90° (y) $\boxed{1}$



Example 5

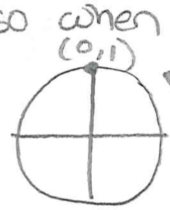
Find the value of $\qquad$ a) cos 360° (x) $\boxed{1}$ $\qquad$ b) sin(2π) (y) $\boxed{0}$ $\qquad$ c) cos(-π) (x) $\boxed{-1}$



Example 6

Solve sinθ = 1 for θ in degrees and radians.

$\sin\theta = y$ so when does $y=1$?



@ 90° or $\pi/2$

$\theta = 90°$ or $\theta = \pi/2$

But coterminal ∠s exist...

$\boxed{\theta = 90 \pm 360n}$ $\qquad$ $\boxed{\theta = \pi/2 + 2\pi n}$

multiple of 2π or 360°

☞ Because the values of sinθ and cosθ repeat, trig functions are Periodic.

Section 7.3 Summary:

The sinθ and cosθ is dependent on θ, not the radius. Using $x^2+y^2=r^2$ and given 2 of the 3 values you can find $\sin\theta = \frac{y}{r}$ and $\cos\theta = \frac{x}{r}$. Remember,

II (-,+)	(+,+) I
III (-,-)	(+,-) IV

(x, y)
(cos, sin)