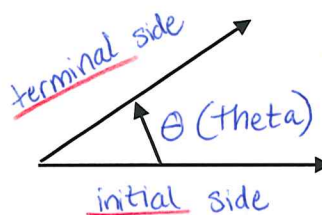


**Section 7.1: Measurement of Angles**

**Essential Question:** How do you convert degrees to radians and vice versa?  
 How do you convert radians to DMS?

**Trigonometry**

- Comes from two Greek words... TRIGONON and METRON meaning "triangle measurement"
- Ancient Egypt used trigonometry to survey land
- Today trigonometry is used for more modern applications such as radio waves



**Measurements**

- Angle: rotation from a point
- Revolution: common unit used to measure very large angles
  - Example: car tire measures rev/min (rpm's)
- Degree: Common unit for small angles
  - 1 Revolution =  $360^\circ$
- Minutes & Seconds: more precise unit of measure
  - 1 degree = 60 minutes ( $60'$ )
  - 1 minute = 60 seconds ( $60''$ )
  - 1 degree = 3600 seconds

**Examples**

1) Given  $24^\circ 18' 20''$  convert to decimal degrees

$$24^\circ + \frac{18}{60} + \frac{20}{3600} = 24.305^\circ$$

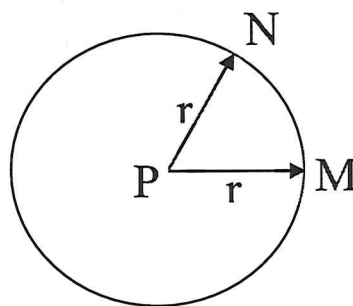
2) Given  $19.7^\circ$  convert to DMS (Degree Minute Second)

$$19.7 = 19 + .7(60) = 19^\circ 42'$$

(multiply decimal by 60)

$\angle NPM$  is a central angle  
 (the center of the circle is the vertex of the angle)

$\widehat{NM}$  is the arc intercepted by  $\angle NPM$   
 (minor arc)



## RADIAN MEASURE

Number of radius units in the length of an arc intercepted by a central angle

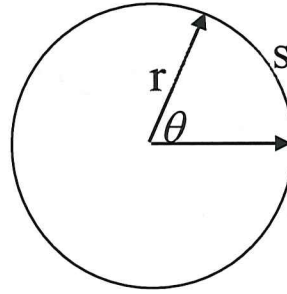
DEGREE MEASURE - amount of turn

RADIAN MEASURE - length

$s$  = arc length

$r$  = radius

$\theta$  = angle measure in radians



Arc Length

$$s = r \cdot \theta$$

\*  $\theta$  is radians

Angle Measure

$$\theta = \frac{s}{r}$$

Note:  $r = \frac{s}{\theta}$

Remember the circumference of a circle is  $C = 2\pi r$  or

$C = d \cdot \pi$  ( $d = \text{diameter} = 2 \cdot \text{radius}$ )

...but when we talk about radian and degree measures we are referring to a unit circle which is a circle with a radius of 1 unit

1 revolution in radians =  $2\pi$

1 revolution in degrees =  $360^\circ$

### CONVERSIONS

$2\pi$  radians =  $360^\circ$  or divide by 2 to get...

$\pi$  radians =  $180^\circ$

1 radian =  $\frac{180^\circ}{\pi}$

or

1 degree =  $\frac{\pi}{180^\circ}$

### Examples

3) Convert  $190^\circ$  to radian measure

$$190^\circ \left( \frac{\pi}{180^\circ} \right) = 3.32$$

4) Convert  $240^\circ$  to radians (leave in terms of  $\pi$ )

$$240 \left( \frac{\pi}{180} \right) = \frac{240\pi}{180} = \frac{24\pi}{18} = \frac{4\pi}{3}$$

5) Convert 2.6 radians to degrees

$$2.6 \left( \frac{180}{\pi} \right) = 149^\circ$$

6) Convert 4.3 radians to DMS

$$4.3 \left( \frac{180}{\pi} \right) = 246.37^\circ$$

$.37(60) = 22.2$   
 $.2(60) = 12$

$246^\circ 22' 12''$

## Commonly Used Radian Measures

Multiples of...  $90^\circ = \frac{\pi}{2}$

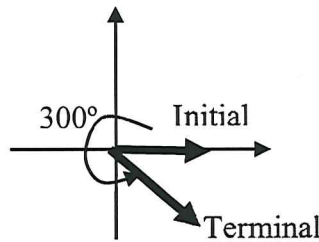
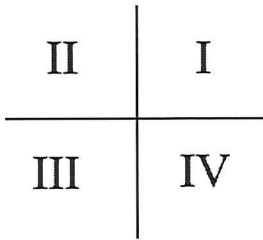
$45^\circ = \frac{\pi}{4}$

$60^\circ = \frac{\pi}{3}$

$30^\circ = \frac{\pi}{6}$

### Standard Position

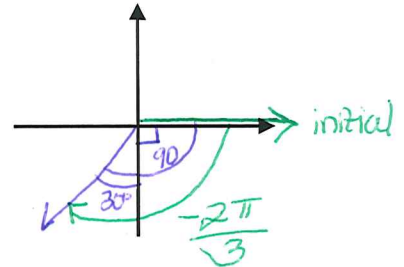
- Vertex is at the origin
- Initial ray on positive x-axis



Counterclockwise

IV Quadrant Angle

Ex:



$-\frac{2\pi}{3}$  Clockwise

3 Q Angle

$$\frac{-2\pi}{3} \left( \frac{180}{\pi} \right) = -120^\circ$$

\*convert to degrees to make easier to graph

### Quadrantal Angles

- Terminal side of angle in standard position lies on an axis
- Always a multiple of  $\pm 90^\circ$  or  $\pm \frac{\pi}{2}$

### Coterminal Angles

- Two angles that have the same terminal side
- There are infinitely many coterminal angles

Ex 8: Give a positive & negative coterminal angle of  $45^\circ$  (degrees and radians)

$$45^\circ + 360 = 405$$

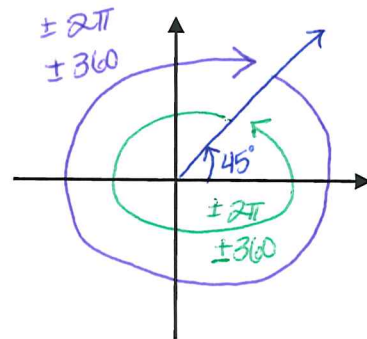
$$45^\circ - 360 = -315^\circ$$

(Add/subtract full circle to end in same position)

$$\frac{\pi}{4} + 2\pi = \frac{\pi}{4} + \frac{8\pi}{4} = \frac{9\pi}{4}$$

$$\frac{\pi}{4} - 2\pi = \frac{\pi}{4} - \frac{8\pi}{4} = \frac{-7\pi}{4}$$

$$45^\circ = \frac{\pi}{4}$$



### Section 7.1 Summary:

$$1 \text{ radian} = \frac{180^\circ}{\pi}$$

$$1 \text{ degree} = \frac{\pi}{180}$$

D → R multiply degree by  $\left(\frac{\pi}{180}\right)$

R → D multiply radian by  $\left(\frac{180}{\pi}\right)$

Radian → DMS multiply radian by  $\left(\frac{180}{\pi}\right)$  then decimal by 60 then multiply new decimal by 60 again