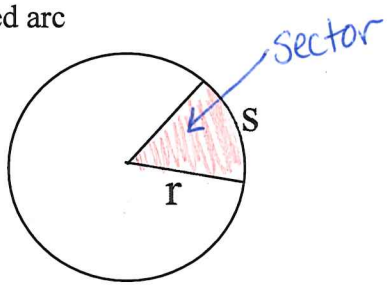
KEY

Section 7.2: Sector Of A Circle

Essential Question: How do you find the arc length and area of a sector?

"Slice of pizza"

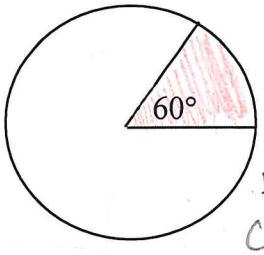
Sector of a Circle: Region bounded by a central angle and the intercepted arc



s = arc length Circumference (C) = $2\pi r$

r = radius Area = πr^2

Given the following circle:



$\frac{60}{360} = \frac{1}{6}$ of the circle

so the arc length = $\frac{1}{6}$ of circumference = $\frac{1}{6} \cdot C$

and the area of the sector = $\frac{1}{6}$ of Area = $\frac{1}{6} \cdot A$

Full Circle = 360°

	Arc Length (s)	***Area*** (K)
If θ is in *DEGREES*:	$s = \frac{\theta}{360} \cdot 2\pi r$	$K = \frac{\theta}{360} \cdot \pi r^2$
If θ is in *RADIANS*:	$s = \theta \cdot r$	$K = \frac{\theta}{2\pi} \cdot \pi r^2 = \frac{\theta}{2} \cdot r^2$

$K = \frac{1}{2}\theta r^2$ → $\frac{1}{2} \cdot \theta \cdot r \cdot r$ (where $s = \theta \cdot r$)

OR $K = \frac{1}{2} \cdot s \cdot r$

Example 1

A sector has a radius of 5 cm (r) and a central angle of .7 radians (θ).
Find its arc length and area.

$s = r\theta$
$= 5(.7)$
$s = 3.5\text{cm}$

$K = \frac{1}{2}\theta r^2$
$= \frac{1}{2}(.7)(5)^2$
$K = 8.75\ \text{cm}^2$

or $K = \frac{1}{2} \cdot r \cdot s$
$= \frac{1}{2}(5)(3.5)$

Example 2

The arc length of a sector is 6cm (s) and the area is 75 cm^2 (K).
What is the radius and central angle?

① $K = \frac{1}{2} \cdot r \cdot s$
$75 = \frac{1}{2} \cdot r \cdot 6$
$75 = 3r$
② $r = 25\text{ cm}$ *need r to find θ

③ $K = \frac{1}{2}\theta r^2$
$75 = \frac{1}{2} \cdot \theta (25)^2$
$75 = 312.5\theta$
④ $\theta = .24$

OR → $s = r \cdot \theta$
$6 = 25 \cdot \theta$
$\frac{6}{25} = \frac{25\theta}{25}$

Example 3 θ *Degrees S K

The central angle is 25° and the arc length is 2.6 cm. What is the area?

① $K = \frac{\theta}{360} \cdot \pi r^2$

② $K = \frac{25}{360} \cdot \pi (\quad)^2$ *need radius

③ $S = \frac{\theta}{360} \cdot 2\pi r$

$2.6 = \left(\frac{25}{360} \cdot 2\pi\right) r$

$2.6 = (.436)\, r$

④ $r = 5.96$

$K = \frac{25}{360} \cdot \pi (5.96)^2$ ← plug in

$K = 7.75\ cm^2$

Example 4

A sector has a perimeter of 16 cm and area of 15cm^2.

What is the radius and arc length?

r S

① $P = r + r + S$

$P = 2r + S$

② $16 = 2r + S$

⑤ $S = 16 - 2r$

③ $K = \frac{1}{2} \cdot r \cdot S$

④ $15 = \frac{1}{2} \cdot r \cdot S$ (2 unknowns → substitution)

⑥ $15 = \frac{1}{2} \cdot r (16 - 2r)$

$15 = 8r - 1r^2$

$r^2 - 8r + 15 = 0$

⑦ $(r-5)(r-3)$

⑧ $r = 5\ cm$ $r = 3\ cm$

Arc length

⑨ $S = 16 - 2(5)$

$S = 6\ cm$

⑩ $S = 16 - 2(3)$

$S = 10\ cm$

APPARENT SIZE: the measure of an angle

Formula to use: $S = r \cdot \theta$

Example 5

The apparent size of a building 2km away is .05 radians.

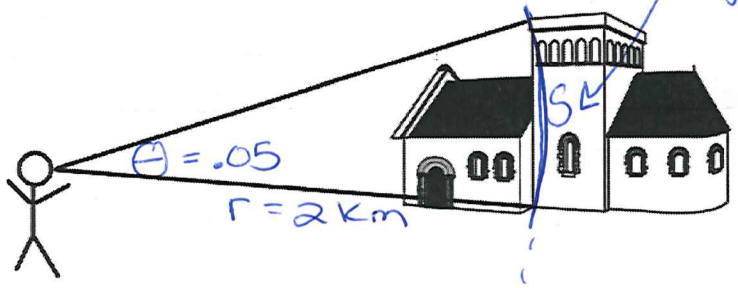
What is its approximate height?

$S = r \cdot \theta$

$S = 2(.05)$

$S = .1\ km$

or 10meters



Section 7.2 Summary:

	Degrees	Radians
Arc Length	$S = \frac{\theta}{360} \cdot 2\pi r$	$S = r \cdot \theta$
Area	$K = \frac{\theta}{360} \cdot \pi r^2$	$K = \frac{1}{2} \theta r^2$ OR $\frac{1}{2} \cdot S \cdot r$