

KEY

Section 7.5: The Other 4 Trigonometric Functions

Essential Question: What are the 6 trig functions and their relationships?

Other Trig Functions

tangent (tan) secant (sec)
cotangent (cot) cosecant (csc)

$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x}$$

$$\csc \theta = \frac{r}{y}$$

$$\sec \theta = \frac{r}{x}$$

$$\cot \theta = \frac{x}{y}$$

RECIPROCALS

$\sec \theta$ and $\cos \theta$ are reciprocals:

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$\csc \theta$ and $\sin \theta$ are reciprocals:

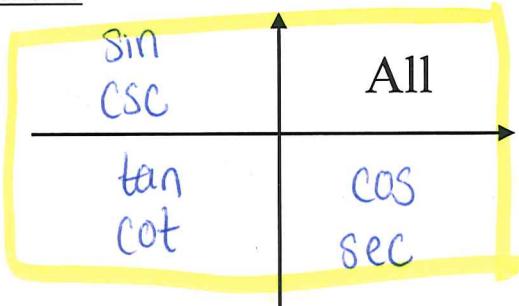
$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sin \theta = \frac{1}{\csc \theta}$$

Since $\sin \theta = y$ and $\cos \theta = x$...

$$\tan \theta = \frac{y}{x} = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{x}{y} = \frac{\cos \theta}{\sin \theta}$$

POSITIVE TRIG VALUESExample 1

Use a calculator to find the following. *Check MODE

a) $\sin 27^\circ$

.4540

b) $\tan 175^\circ$

-0.0875

c) $\sec 42^\circ$

$= \frac{1}{\cos 42^\circ}$

$= 1.3456$

d) $\cot 7$

$= \frac{1}{\tan 7}$

$= 1.1475$

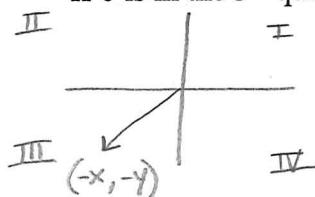
e) $\csc(-1)$

$= \frac{1}{\sin(-1)}$

$= -1.1884$

Example 2

If θ is in the 3rd quadrant and $\tan \theta = 5/12$ find the other five trig functions.



$$\tan \theta = \frac{5}{12} = \frac{y}{x} \quad y = -5 \quad x = -12$$

$$x^2 + y^2 = r^2 \\ (-12)^2 + (-5)^2 = r^2 \\ 169 = r^2 \rightarrow r = 13$$

$$\cot \theta = \frac{12}{5} \quad \cos \theta = -\frac{12}{13} \\ \sin \theta = -\frac{5}{13} \quad \csc \theta = -\frac{13}{5} \\ \sec \theta = -\frac{12}{5}$$

Example 3

Express each of the following in term of a reference angle.

\sin	All
\csc	\cos
\tan	\cot
\sec	\sec

a) $\sin 190^\circ = \boxed{-\sin 10^\circ}$

b) $\sec(-100^\circ) = \boxed{-\sec 80^\circ}$

c) $\tan 300^\circ = \boxed{-\tan 60^\circ}$

d) $\csc 125^\circ = \boxed{\csc 55^\circ}$

e) $\cos(-72^\circ) = \boxed{\cos 72^\circ}$

Example 4

Give the value of x (in radians) for which $\sec(x)$ is: Change 'x' to ' θ '

a) 1

$$\sec \theta = 1$$

$$\cos \theta \left(\frac{1}{\cos \theta}\right) = (1) \cos \theta$$

$$1 = \cos \theta$$

when $\theta = 2\pi \cdot n$ multiples of 2π

b) 0

$$\sec \theta = 0$$

$$\cos \theta \left(\frac{1}{\cos \theta} = 0\right) \cos \theta$$

$$1 \neq 0$$

Not possible

No Solution

c) undefined = $\sec \theta$

$$\frac{1}{\cos \theta} = \text{undef.}$$

if $\cos \theta = 0$ then $\frac{1}{0} = \text{und.}$

$$\theta = \frac{\pi}{2} \text{ & } \frac{3\pi}{2}$$

$$\theta = \frac{\pi}{2} + \pi n$$

Example 5

Give the value of x (in radians) for which $\tan(x)$ is:

a) 0

$$\tan \theta = 0$$

$$\cos \theta \left(\frac{\sin \theta}{\cos \theta} = 0\right) \cos \theta$$

$$\sin \theta = 0$$

$$\theta = \pi \text{ & } 2\pi \rightarrow \theta = \pi n$$

b) 1

$$\tan \theta = 1$$

$$\cos \theta \left(\frac{\sin \theta}{\cos \theta} = 1\right) \cos \theta$$

$$\sin \theta = \cos \theta$$

$$\theta = \pi/4 \text{ & } 5\pi/4$$

$$\theta = \pi/4 + \pi n$$

c) undefined

$$\tan \theta = \text{und.}$$

$$\frac{\sin \theta}{\cos \theta} = \text{undefined}$$

when $\cos \theta = 0$

$$\theta = \frac{\pi}{2} + \pi n$$

Example 6

Given $\tan \theta = -\frac{3}{4}$ and $0 < \theta < \pi$, find the other trig values.

$$\tan \theta = -\frac{3}{4} = \frac{y}{x}$$

$$x = -4$$

$$y = 3$$

$$r = ?$$

$$x^2 + y^2 = r^2$$

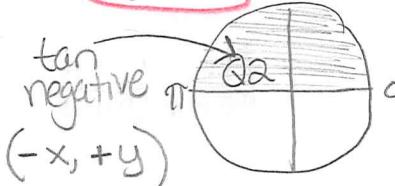
$$(-4)^2 + (3)^2 = r^2$$

$$16 + 9 = r^2$$

$$r^2 = 25$$

$$r = 5$$

* radius always positive



$$\cot \theta = \frac{x}{y} = -\frac{4}{3}$$

$$\cos \theta = \frac{x}{r} = -\frac{4}{5} \quad \sec \theta = \frac{5}{4}$$

$$\sin \theta = \frac{y}{r} = \frac{3}{5} \quad \csc \theta = \frac{5}{3}$$

Example 7 → No Decimals

Find the exact values.

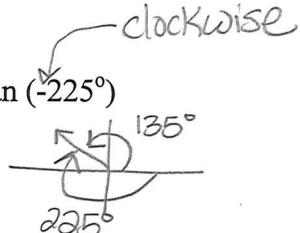
a) $\sec 150^\circ$

↓ reciprocal of
 $\cos 150^\circ$

$$\cos(150^\circ) = -\frac{\sqrt{3}}{2}$$

$$\sec 150^\circ = \frac{-2}{\sqrt{3}} = \boxed{\frac{-2\sqrt{3}}{3}}$$

b) $\tan(-225^\circ)$



$$\begin{aligned} &= \tan 135^\circ = \frac{y}{x} \\ &= \frac{\sqrt{2}/2}{-\sqrt{2}/2} = \boxed{-1} \end{aligned}$$

c) $\sec 180^\circ$

Reciprocal $\cos \theta$

$$\cos(180^\circ) = -1$$

$$\begin{aligned} \sec 180^\circ &= \frac{1}{-1} \\ &= \boxed{-1} \end{aligned}$$

d) $\tan 90^\circ = \frac{y}{x}$

@90° (0,1)
x y

$$= \frac{1}{0} = \boxed{\text{undefined}}$$

Section 7.5 Summary: