KEY

Section 8.4: Relationships Among the FUNctions
Essential Question: How do you simplify a trignometric expression?

When simplifying an expression the final answer is often in terms of a trignometric function or an integer.

Here are some ***HINTS*** to help you out:

- Cancel any terms if possible
- Using trig identities, simplify
- Write in terms of sine and cosine
- Do the indicated operation (add, subtract, multiply...)
- Multiply numerator and denominator by a LCD
- Factor, then cancel factors to simplify

Ex1

$\sin\theta \cdot \sec\theta \cdot \cot\theta$

$\cancel{\sin} \cdot \dfrac{1}{\cancel{\cos}} \cdot \dfrac{\cancel{\cos}}{\cancel{\sin}} = \boxed{1}$

reciprocal

* cancel numerator/denominator

Ex2

$(1 - \sin\theta)(1 + \sin\theta)$ FOIL

$1 + \cancel{\sin} - \cancel{\sin} - \sin^2$

$\underbrace{1 - \sin^2}_{\text{identity}} = \boxed{\cos^2\theta}$

Ex3

$\sin\theta \cdot \cot\theta$

$\cancel{\sin} \cdot \dfrac{\cos}{\cancel{\sin}} = \boxed{\cos\theta}$

Ex4

$\dfrac{\cos\theta \cdot \sin\theta}{1 - \sin^2\theta}$

$\dfrac{\cancel{\cos} \cdot \sin}{\cos^{\cancel{2}}} = \dfrac{\sin}{\cos} = \boxed{\tan\theta}$

* cancel one cos

Ex5

$1 + \dfrac{\cos^2\theta}{1 - \cos^2\theta}$ ← identity

$1 + \dfrac{\cos^2}{\sin^2} = \boxed{1 + \cot^2\theta}$

no identity

Ex6

$\dfrac{\tan x + \cot x}{\sec^2 x} = \dfrac{\dfrac{\sin}{\cos}\dfrac{(\sin)}{(\sin)} + \dfrac{\cos}{\sin}\dfrac{(\cos)}{(\cos)}}{\dfrac{1}{\cos^2}}$ * to add common denominator

$= \dfrac{\dfrac{\overbrace{\sin^2 + \cos^2}^{\text{identity}}}{\sin \cdot \cos}}{\dfrac{1}{\cos^2}} = \left(\dfrac{1}{\sin\cos}\right)\left(\dfrac{\cos^2}{1}\right)$ ← mult by reciprocal

$\left(\dfrac{1}{\sin \cdot \cancel{\cos}}\right)\left(\dfrac{\cos^{\cancel{2}}}{1}\right) = \dfrac{\cos}{\sin} = \boxed{\cot\theta}$

Ex7

$$\frac{\cot x - \tan x}{\sin x \cdot \cos x}$$

$$\frac{\frac{(\cos)\cos}{(\cos)\sin} - \frac{\sin(\sin)}{\cos(\sin)}}{\sin\cdot\cos}$$ * common denominator

$$\frac{\frac{\cos^2-\sin^2}{\sin\cdot\cos}}{\frac{\sin\cdot\cos}{1}} = \left(\frac{\cos^2-\sin^2}{\sin\cdot\cos}\right)\left(\frac{1}{\sin\cdot\cos}\right)$$

* mult. by den. rec.

seperate to cancel

$$= \frac{\cos^2-\sin^2}{\sin^2\cdot\cos^2} = \frac{\cancel{\cos^2}}{\sin^2\cdot\cancel{\cos^2}} - \frac{\cancel{\sin^2}}{\cancel{\sin^2}\cdot\cos^2}$$

$$\frac{1}{\sin^2} - \frac{1}{\cos^2} = \boxed{\csc^2\theta - \sec^2\theta}$$

Ex8

$$\frac{1-\sin^2\theta}{1+\cot^2\theta} = \frac{\cos^2}{\csc^2} = \frac{\cos^2}{\frac{1}{\sin^2}}$$

$$= (\cos^2)\left(\frac{\sin^2}{1}\right)$$

$$= \boxed{\cos^2\theta\cdot\sin^2\theta}$$

Ex9

$$\frac{\tan^2\theta}{1+\tan^2\theta}$$

$$\frac{\tan^2}{\sec^2} = \frac{\frac{\sin^2}{\cos^2}}{\frac{1}{\cos^2}}$$

$$= \left(\frac{\sin^2}{\cancel{\cos^2}}\right)\left(\frac{\cancel{\cos^2}}{1}\right) = \boxed{\sin^2\theta}$$

* mult by reciprocal

Ex10

FOIL

$$\frac{(\tan)\tan x}{(\tan)\,1+\sec x} + \frac{1+\sec x\,(1+\sec)}{\tan x\,(1+\sec)}$$ * common denominator

$$= \frac{\overbrace{\tan^2+1}^{\text{identity}}+2\sec+\sec^2}{(\tan)(1+\sec)}$$

$$= \frac{\sec^2+2\sec+\sec^2}{(\tan)(1+\sec)} = \frac{2\sec^2+2\sec}{(\tan)(1+\sec)}$$

$$= \frac{2\sec(\cancel{\sec+1})}{(\tan)(\cancel{1+\sec})} = \frac{2\sec}{\tan} = \frac{\frac{2}{\cos}}{\frac{\sin}{\cos}}$$

$$= \left(\frac{2}{\cancel{\cos}}\right)\left(\frac{\cancel{\cos}}{\sin}\right) = \frac{2}{\sin} = \boxed{2\cdot\csc\theta}$$

Section 8.4 Summary:

To simplify a trig expression cancel at any time possible. Change parts using identities, but if none exist then change the expression into terms of sine & cosine. Do the indicated operation and cancel until no identities or cancelations remain.