

Section 17.3: Variability

Essential Question:

How do you calculate standard deviation and z-scores?

Definitions

Statistic is a number that describes some characteristic of a set of data**Mean, Median, Mode** describe the center of a set of data**Range and Interquartile Range** describe the spread of the data about the center or called Measure of Dispersion (how much data varies)

Example 1:

The following 3 classes have the given test scores.

Class 1
50, 50, 50, 50, 50Class 2
40, 50, 50, 50, 60Class 3
10, 40, 50, 60, 90

Mean = 50

Mean = 50

Mean = 50

Range = $50 - 50 = 0$ Range = $60 - 40 = 20$ Range = $90 - 10 = 80$ OTHER MEASURES OF DISPERSION: Variance and Standard Deviation**Variance**: Written as s^2 or σ^2 (sigma squared)Data: $x_1, x_2, x_3, x_4, \dots, x_n$ ($n = \#$ of items)Mean = $\bar{x} = \frac{\text{sum of data}}{n}$

$$\sigma^2 = s^2 = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + (x_3 - \bar{x})^2 + \dots}{n}$$

Standard Deviation: Written as s or σ

*It is how much items are dispersed around the mean - NOT an average!

$$\sigma = \sqrt{\text{variance}} = \sqrt{\sigma^2}$$

Example 2:

Five pairs of shoes cost \$10, \$20, \$30, \$40 and \$50.

$$\text{Mean} = \bar{x} = \frac{10 + 20 + 30 + 40 + 50}{5} = \frac{150}{5} = 30 \rightarrow \text{average shoe pair is } \$30$$

$$\text{Variance} = \sigma^2 = \frac{(10-30)^2 + (20-30)^2 + (30-30)^2 + (40-30)^2 + (50-30)^2}{5} = \frac{1000}{5} = 200$$

$$\text{Standard Deviation} = \sqrt{\text{variance}} = \sqrt{200} \approx \$14.14$$

*not an average
* a measure of dispersion30 - 14.14 = \$15.86
30 + 14.14 = \$44.14
most shoes cost between

Example 3:Find the standard deviation for 1, 7, 9, 15

$$\bar{X} = \frac{1+7+9+15}{4} = \frac{32}{4} = 8$$

$$\begin{aligned} \text{Variance} &= \sigma^2 \\ &= \frac{(1-8)^2 + (7-8)^2 + (9-8)^2 + (15-8)^2}{4} = \frac{100}{4} = 25 \end{aligned}$$

$$\text{st. dev} = \sigma = \sqrt{25} = \boxed{5}$$

Example 4:Find the mean and standard deviation from the table of data.

x_i = item of data	4	6	10
f_i = frequency	2	5	3

of distinct data items = 3

of total data items = 10

$$\bar{X} = \frac{2(4) + 5(6) + 3(10)}{10} = \frac{68}{10} = 6.8$$

$$\sigma^2 = \frac{2(4-6.8)^2 + 5(6-6.8)^2 + 3(10-6.8)^2}{10} = \frac{49.6}{10} = 4.96$$

$$\begin{aligned} \text{st. dev} &= \sqrt{\text{var}} \\ \sigma &= \sqrt{4.96} \\ \sigma &= \boxed{2.22} \end{aligned}$$

Standard Value (z) of a piece of data

$$\text{Standard Value} = \frac{\text{data item} - \text{mean}}{\text{standard deviation}}$$

OR

$$z = \frac{X - \bar{X}}{\sigma}$$

Z give you the number of standard deviations between an item of the data and the mean
 (Also called z-score or relative score)

Example 5:

Use data from example 2 to find the standard value if you had a score of...

$$1, 7, 9, 15 \quad \bar{X} = 8 \quad \sigma = 5$$

Score of 1

$$z = \frac{1-8}{5} = \boxed{-1.4}$$

Score of 7

$$z = \frac{7-8}{5} = \boxed{-0.2}$$

Score of 9

$$z = \frac{9-8}{5} = \boxed{0.2}$$

Score of 15

$$z = \frac{15-8}{5} = \boxed{1.4}$$

Section 17.3 Summary:

Standard Deviation measures dispersion (spread)

① Find mean = $\bar{X} = \frac{\text{sum of data}}{\text{\# of items}}$

② Calculate variance = $s^2 = \sigma^2 = \frac{(\text{data}_1 - \bar{X})^2 + (\text{data}_2 - \bar{X})^2 + (\text{data}_3 - \bar{X})^2 + \dots}{n}$
 $n \leftarrow \# \text{ of items}$

③ standard dev. = σ or $S = \sqrt{\text{variance}}$