KEY

Section 4.3: Reflecting Graphs and Symmetry

Essential Question:
How do you reflect over and test for symmetry to: x-axis, y-axis, y = x

1) Reflection over the x-axis

- y = -f(x) is a reflection of y = f(x) over the x-axis
- (x , -y) is the reflected coordinate of (x, y)

Examples

y = x^2 − 4 OR y = 2x + 1

$y = -(x^2-4) \rightarrow y = -x^2+4$ $y = -(2x+1) \rightarrow y = -2x-1$

2) Entire graph above the x-axis

- Absolute Value (x,y) becomes (x, |y|)

Example

y = x^2 − 5 becomes $y = |x^2-5|$ ("v" shape if linear)

3) Reflection over the y-axis

- y = f(-x) is a reflection of y = f(x) over the y-axis
- (-x , y) is the reflected coordinate of (x, y)

Examples

y = 2^x OR $y = (x+2)^2$

$y = 2^{-x} = \frac{1}{2^x}$ $y = (-x+2)^2$

4) Reflection over the line y = x

- Interchange the x and y
- (y , x) is the reflected coordinate of (x, y)

Examples

y = 3x + 1 OR $y = x^3$

$x = 3y+1$ $x = y^3$

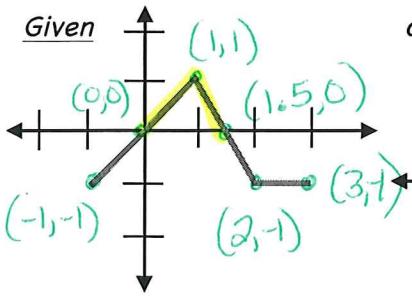
$\frac{x-1}{3} = \frac{3y}{3}$ $\sqrt[3]{x} = \sqrt[3]{y^3}$ $y = \sqrt[3]{x}$

$y = \frac{1}{3}x - \frac{1}{3}$

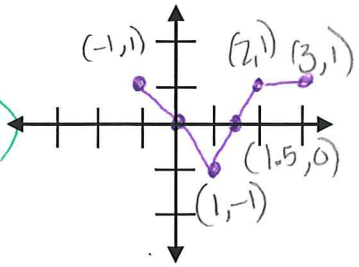
Example 1

Given the graph of y = f(x) sketch the graph of...

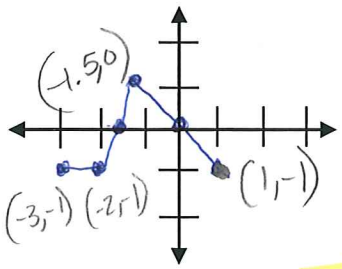
Given



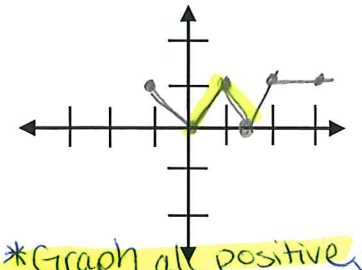
a) y = -f(x) (x, -y) *change y



b) y = f(-x) (-x, y) *change x



c) y = |f(x)| (x, |y|)



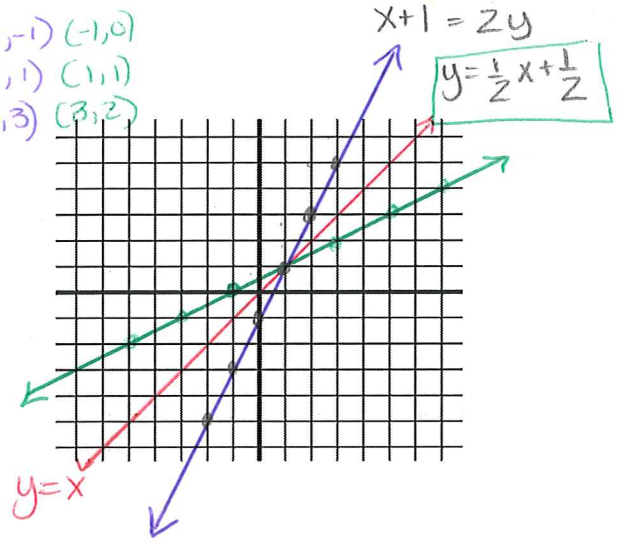
*Graph all positive y-values first (all above x-axis)

Example 2

Sketch the graph of the equation and the reflection of the graph over y = x.
Give the equation of the new graph.

a) * y = 2x − 1 → new: $x = 2y - 1$, $x + 1 = 2y$, $y = \frac{1}{2}x + \frac{1}{2}$

(0,−1) (−1,0) (1,1) (1,1) (2,3) (3,2)



b) * $y = x^2 - 4x$

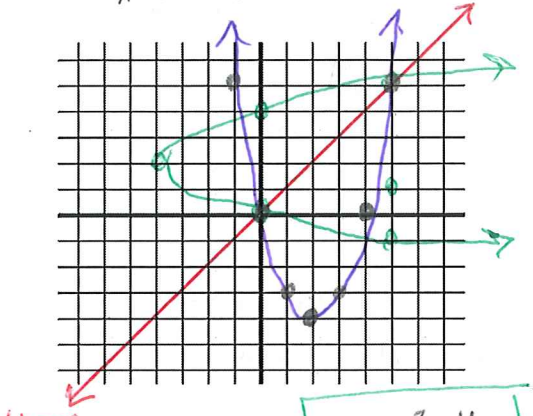
$\frac{-b}{2a} = \frac{4}{2} = 2$

$x(x-4)$

$x = 0$ $x = 4$

(2,−4)

x	y
0	0
4	0
2	−4
−1	5
5	5



$x = y^2 - 4x$

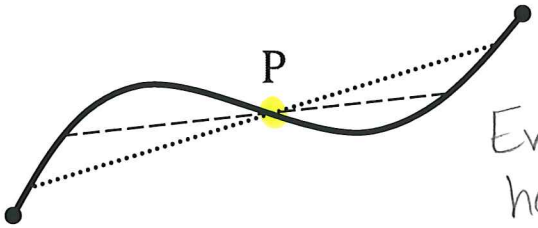
*New

x	y
0	0
0	4
−4	2
5	−1
5	5

Point of Symmetry (POS)

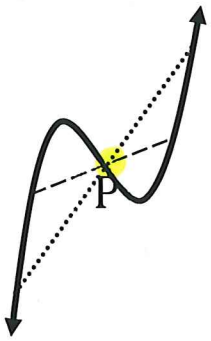
A point P is called POS of a graph if you can pair the points so P is the midpoint of the segment joining each pair of points.

Examples



Every cubic equation has a POS (x,y)

$x = \frac{-b}{3a}$ → plug x in solve for y



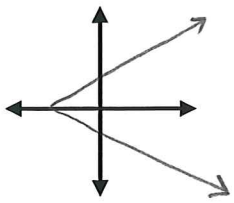
Axis of Symmetry (AOS) or Line of Symmetry (LOS)

- AOS $x = \frac{-b}{2a}$ (vertical line)
- splits graph in half

How to Test for Symmetry of a Graph

Symmetric to the x-axis

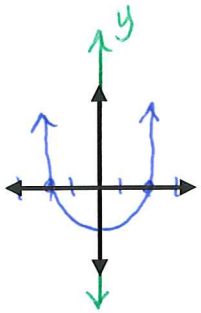
- (x,y) and (x,−y) are on the graph
- Test: leave x alone and substitute −y for y
- Do you get an equivalent equation?



KEY

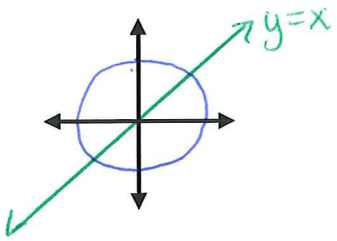
Symmetric to the y-axis

- (x,y) and (−x,y) on graph
- Test: leave y alone and substitute −x for x
- Do you get an equivalent equation?



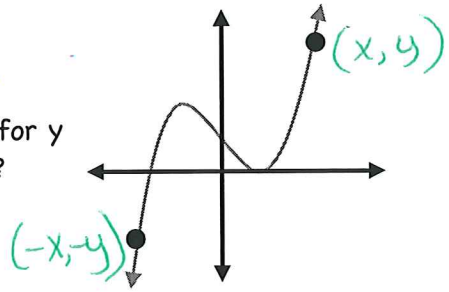
Symmetric to the line y = x

- (x,y) and (y,x) on graph
- Test: Interchange x and y
- Do you get an equivalent equation?



Symmetric to the origin

- (x,y) and (−x,−y) on graph
- Test: Substitute −x for x and −y for y
- Do you get an equivalent equation?



Examples #3 – 6

Test for symmetry with...

a) the x-axis $y \to -y$ b) the y-axis $x \to -x$ c) line y = x $x \leftrightarrow y$ d) the origin $x \to -x$ AND $y \to -y$

#3

$x^4 + y^4 = 1$

a) $x^4+(-y)^4=1$
$x^4+y^4=1$ yes

b) $(-x)^4+y^4=1$
$x^4+y^4=1$ yes

c) $y^4+x^4=1$ yes
(commutative)

d) $(-x)^4+(-y)^4=1$
$x^4+y^4=1$ yes

#4

$xy^3 = 1$

a) $x(-y)^3=1$
$-xy^3=1$ NO

b) $(-x)y^3=1$
$-xy^3=1$ NO

c) $y\cdot x^3=1$ NO

d) $(-x)(-y)^3=1$
4 neg = pos
$xy^3=1$ yes

#5

$x(x + y) = 1$

$x^2+xy=1$

a) $x^2+x(-y)=1$
$x^2-xy=1$ NO

b) $(-x)^2+(-x)y=1$
$x^2-xy=1$ NO

c) $y^2+yx=1$ NO

d) $(-x)^2+(-x)(-y)=1$
$x^2+xy=1$ yes

#6

$y = x^2 + 7$

a) $-y=x^2+7$ NO

b) $y=(-x)^2+7$
$y=x^2+7$ yes

c) $x=y^2+7$ NO

d) $-y=(-x)^2+7$
$-y=x^2+7$ NO

Example 7

Find the AOS for

$y = x^2 - 8x - 7$

AOS: $x=\frac{-b}{2a}=\frac{8}{2(1)}=4$

$x=4$

$x=\#$ ↕ vertical line

Example 8 ← point (x,y)

Find the POS for

$y = x^3 - 6x^2 + 5x + 7$

$x=\frac{-b}{3a}=\frac{6}{3(1)}=2$

$y=(2)^3-6(2)^2+5(2)+7$
$8-24+10+7=1$

$(2,1)$

Section 4.3 Summary:

	x-axis	y-axis	$y = x$	Above x-axis (abs. value)
Reflect	$(x,y) \rightarrow (x,-y)$	$(x,y) \rightarrow (-x,y)$	$(x,y) \rightarrow (y,x)$	$(x,y) \rightarrow (x,\lvert y\rvert)$
Test for Symmetry	Substitute $-y$ for y same?	Sub $-x$ for x same?	Sub x for y AND y for x same?	N/A