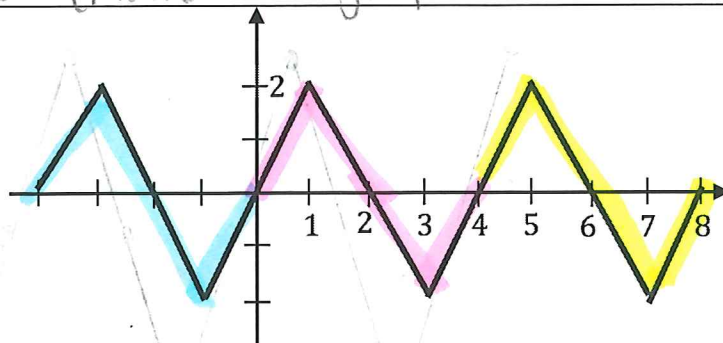


**Section 4.4: Periodic Functions; Stretching & Translating Graphs**

Essential Question: ① Find the period and amplitude of a function  
 ② How do you stretch and shrink a graph horizontally & vertically?  
 ③ How do you translate a graph?



This graph is Periodic; it has a positive period P

**Fundamental Period** is the smallest period of a function

A function  $f$  is periodic if  
 $f(x) = f(x + mp)$

$p$  = value of fundamental period

$m$  = number of periods within a value

**Example 1**

Find the fundamental period of  $f(x)$ .

The graph repeats every four units

$P=4$

**Example 2**

Find  $f(103)$

$\frac{103}{4} = 25\frac{3}{4}$   $f(103) = f(3) = -2$

**Example 3**

Find  $f(106)$

$\frac{106}{4} = 26\frac{2}{4}$   $f(106) = f(2) = 0$

**AMPLITUDE** is the average of the maximum and minimum values of a periodic function

$A = \frac{\text{Max} - \text{min}}{2}$

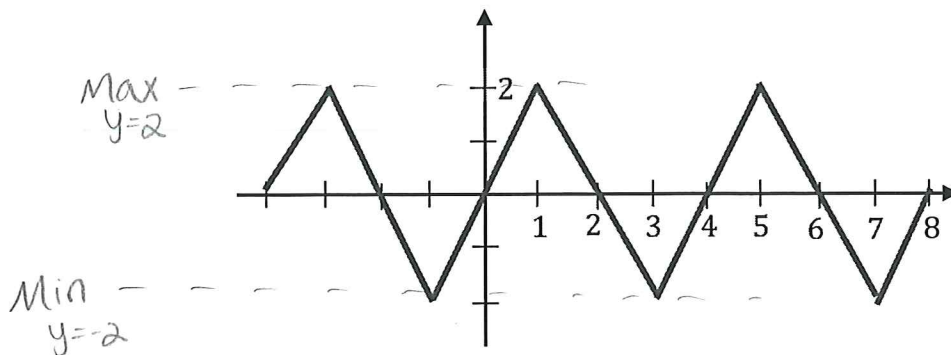
M: maximum y-value

m: minimum y-value

**Example 4**

Find the amplitude of  $f(x)$

$A = \frac{2 - (-2)}{2} = \frac{4}{2} = 2$



\* Remember  $f(x) = y$

**STRETCHING AND SHRINKING A GRAPH**

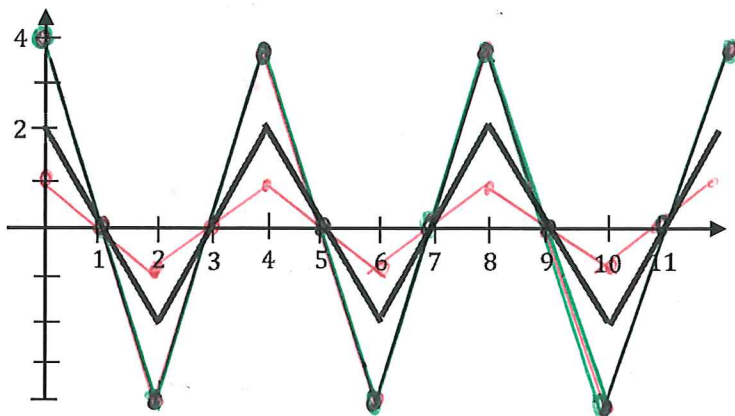
**Vertical stretch or shrink:**  $y = c \cdot f(x)$

$P = 4$

$A = 2$

$y = 2f(x)$   $P = 4$   
 $(x, 2 \cdot y)$   $A = 4$

$y = \frac{1}{2} f(x)$   $P = 4$   
 $(x, \frac{1}{2} \cdot y)$   $A = 1$



If  $|c| > 1$  the graph is vertically stretched

If  $0 < |c| < 1$  the graph is vertically shrunk

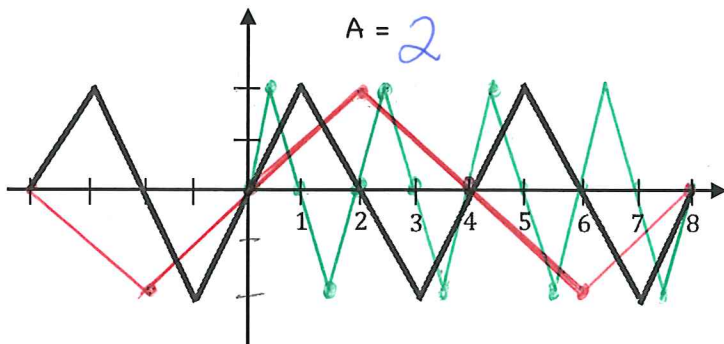
Note: changes amplitude, NOT period

**Horizontal stretch or shrink:**  $y = f(c \cdot x)$   
 (inverse operation)

$P = 4$

$A = 2$

$y = f(2x)$ $(\frac{1}{2} \cdot x, y)$ $P = 2$ $A = 2$	original	$f(2x)$	$f(\frac{1}{2}x)$
	(0, 0)	(0, 0)	(0, 0)
	(1, 2)	( $\frac{1}{2}$ , 2)	(2, 2)
	(2, 0)	(1, 0)	(4, 0)
	(3, -2)	( $\frac{3}{2}$ , -2)	(6, -2)
$y = f(\frac{1}{2}x)$ $(2 \cdot x, y)$ $P = 8$ $A = 2$	(4, 0)	(2, 0)	(8, 0)
	(5, 2)	( $\frac{5}{2}$ , 2)	(10, 2)
	(6, 0)	(3, 0)	(12, 0)



If  $|c| > 1$  the graph is horizontally shrunk (squeezed)

If  $0 < |c| < 1$  the graph is horizontally stretched

Note: changes period, NOT amplitude

Changing Period and Amplitude of a Periodic Function

If a periodic function  $f$  has a period  $P$  and amplitude  $A$  ...

Then  $y = c \cdot f(x)$  has period  $\underline{P}$  and amplitude  $\underline{c \cdot A}$

Then  $y = f(cx)$  has period  $\underline{\frac{1}{c} \cdot P}$  and amplitude  $\underline{A}$

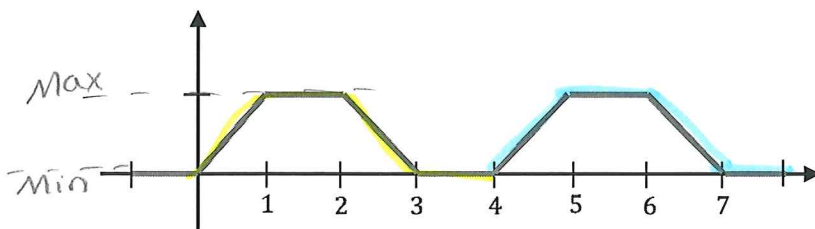
For examples #5 - 7 use the graph below:

**Example 5**

Find the period and amplitude of  $f(x)$

$P = 4$

$A = \frac{1-0}{2} = \frac{1}{2}$



**Example 6**

Find the period and amplitude of

$y = 2f(x) \rightarrow 2 \cdot y$  changes amp.

$P = 4$

$A = 2 \cdot (\frac{1}{2}) = 1$

**Example 7**

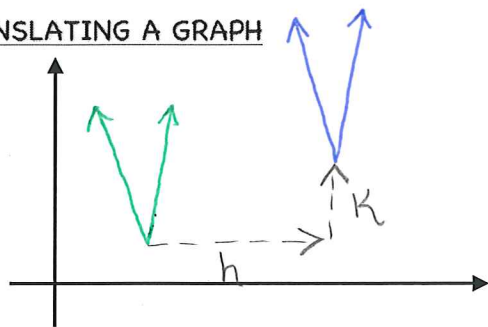
Find the period and amplitude of

$y = f(\frac{1}{2}x) \rightarrow \frac{2}{1} \cdot x$  changes period

$P = \frac{2}{1}(4) = 8$

$A = \frac{1}{2}$

TRANSLATING A GRAPH



$y - k = f(x - h)$  is the same as:

$y = f(x - h) + k$

Horizontal Movement

Vertical Movement

Remember "opposite" of  $h$

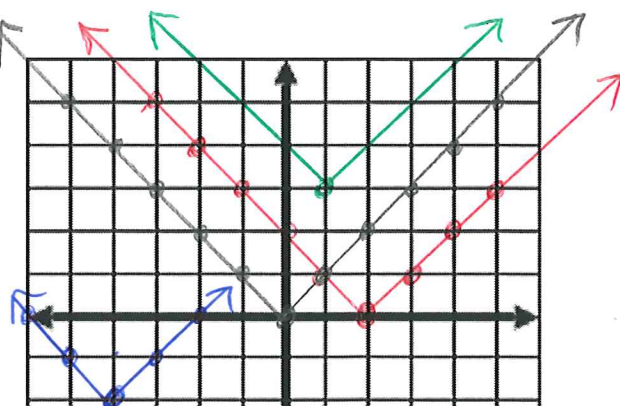
**Example 8**

Graph  $y = |x|$ . Vertex  $(0,0)$   $a=1 \rightarrow$  slope is  $\pm 1$

$y = |x-2|$  right 2 Vertex  $(2,0)$

$y = |x-1| + 3$  Vertex  $(1,3)$  1 right 3 up

$y = |x+4| - 2$  Vertex  $(-4,-2)$  4 left 2 down



Note: When doing multiple translations think in terms of order of operations

Reminder:  $(x - h)^2 + (y - k)^2 = r^2$

A circle with center  $(h, k)$  and radius =  $r$

Section 4.4 Summary:

- ① Period is how often the graph repeats (P)  
Amplitude is the change in vertical movement (average)  
$$A = \frac{(\text{max } y\text{-value}) - (\text{min } y\text{-value})}{2}$$

②

	Stretch	Shrink/Squeeze
Vertical	$y = c \cdot f(x)$ if $ c  > 1$ $(x, c \cdot y)$ changes amplitude	$y = c \cdot f(x)$ if $0 <  c  < 1$ $(x, c \cdot y)$ changes amplitude
Horizontal	$y = f(c \cdot x)$ $(\frac{1}{c} \cdot x, y)$ if $0 <  c  < 1$ Changes Period	$y = f(c \cdot x)$ $(\frac{1}{c} \cdot x, y)$ if $ c  > 1$ changes period

- ③ Translate / Shift

$$y = f(x - h) + k$$

$(h, k)$  vertex

$-h$ right	$+h$ left
$-k$ down	$+k$ up