Algebra III: *Hutschenreuter*
Semester I
Review

Name KEY

Hour _____ Date _____

1. Solve algebraically. $y = -2x + 5$, $y - 3x = 0$

*Substitute

$-2x + 5 - 3x = 0$

$-5x = -5$

$x = 1$

$y = -2(1) + 5$

$-2 + 5$

$y = 3$

*write as a coordinate

$(1, 3)$

2. Find the coordinates of A if the midpoint of $\overline{AB}$ is (2, 3) and B = (1, 8).

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{x+1}{2}, \frac{y+8}{2}\right) = (2, 3)$

$\frac{x+1}{2} = 2$ $\quad x+1=4$ $\quad x=3$

$\frac{y+8}{2} = 3$ $\quad y+8=6$ $\quad y=-2$

$A(3, -2)$

3. Find the distance and the midpoint of the following coordinates.

$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$ → $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

a) M(3,3) A(15,12)

$MA = \sqrt{(3-12)^2 + (3-15)^2}$

$= \sqrt{(-9)^2 + (-12)^2}$

$= \sqrt{81+144} = \sqrt{225} = 15$ un

$\left(\frac{3+15}{2}, \frac{3+12}{2}\right)$

$(9, 15/2)$

b) T(-2,-1) H(4,9)

$TH = \sqrt{(4-2)^2 + (9-1)^2}$

$= \sqrt{6^2+10^2}$

$= \sqrt{36+100} = \sqrt{136} \approx 11.7$ un

$\left(\frac{-2+4}{2}, \frac{-1+9}{2}\right)$

$(1, 4)$

4. Find the slope of $\overline{MA}$ and $\overline{TH}$.

$m = \frac{y_1-y_2}{x_1-x_2}$

$m(MA) = \frac{12-3}{15-3} = \frac{9}{12} = \frac{3}{4}$

$m(TH) = \frac{9-1}{4-2} = \frac{10}{6} = \frac{5}{3}$

5. State which lines are perpendicular, parallel, or neither.

// same slope

⊥ opposite reciprocals

A) $3y = 5x - 3$

$y = \frac{5}{3}x - 1$

$m = 5/3$

B) $-5y + 10 = 3x$

$-5y = 3x - 10$

$y = -\frac{3}{5}x + 2$

$m = -\frac{3}{5}$

C) $6y = \frac{10}{6}x + \frac{1}{6}$

$y = \frac{5}{3}x + \frac{1}{6}$

$m = \frac{5}{3}$

$A \parallel C$

$A \perp B$

$B \perp C$

6. Write the slope-intercept form of the equation of the line passing through (5, 0) and having slope of $\frac{1}{7} = m$

x_1, y_1

point-slope form

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{1}{7}(x - 5)$ → $y = \frac{1}{7}x - \frac{5}{7}$

7. There is a correlation between the height of human and their ulna bone. Aiden is 66" tall and his ulna bone is 11". Blake is 36" tall and his ulna bone is 6". Write a linear equation relating ulna length (y), to the height (x).

A(66, 11)

B(36, 6)

$m = \frac{11-6}{66-36} = \frac{5}{30} = \frac{1}{6}$

$y - 11 = \frac{1}{6}(x - 66)$

$y - 11 = \frac{1}{6}x - 11$

$+11 \quad +11$

$y = 1/6\,x$

* $i^2 = -1$

8. Simplify.

a) $(5-8i)-(10-3i)$

$5-8i-10+3i$

$-5-5i$

b) $(1+i)^2$

$(1+i)(1+i)$

$1+i+i+i^2 = 1+2i+-1$

$= 2i$

9. What are the roots of:

$x^2 + 5x + 6 = 0$

$(x+3)(x+2)=0$

$x=-3$ $x=-2$

10. What are the solutions of:

$x^2 + 2x + 5 = 0$

Not factorable

$$\frac{-2\pm\sqrt{(2)^2-4(1)(5)}}{2(1)} = \frac{-2\pm\sqrt{4-20}}{2} = \frac{-2\pm\sqrt{-16}}{2} \rightarrow \frac{-2\pm 4i}{2}$$

$-1\pm 2i$

11. What is the axis of symmetry and vertex of: $y = -2x^2 + 10x - 4$.

AOS $x = \frac{-b}{2a} = \frac{-10}{2(-2)} = \frac{-10}{-4} = \frac{5}{2} = 2.5$

$x=2.5$

$y = -2(-2.5)^2+10(2.5)-4$

$y = 8.5$

$(2.5, 8.5)$

12. When a ball is thrown upward, its approximate height in meters t seconds later is given by $h(t) = 36 + 24t - 5t^2$. When will the ball reach its highest elevation? What is the height at that particular time?

Vertex (max) (time, height) t h

$t = \frac{-b}{2a} = \frac{-24}{2(-5)} = \frac{24}{10} = 2.4$ sec (t)

$h(2.4)$

$= 64.8$ ft (h)

13. Which of the following is a root of $2x^4 - x^3 - 4x^2 + 8x - 5 = 0$?

$f(1)=0$

a) 1 b) 2 c) 3 d) 4

plug in → if $=0$ then it's a root

14. Which of the following is a factor of $x^3 - 5x^2 + 11x - 10$?

No remainder, $f(x)=0$ to be a factor

a) $x+2$ $x=-2$ b) $x-2$ $x=2$ c) $x+1$ $x=-1$ d) $x-3$ $x=3$

$f(2)=0$

15. $(x + 3)$ is a factor of which of the following?

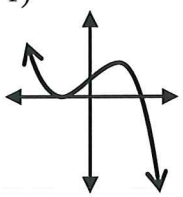
$x=-3$

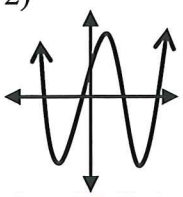
a) $x^3 + 2x + 15$ $f(-3)=-18$ b) $-x^3 + 4x + 15$ $f(-3)=30$

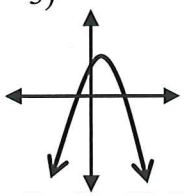
c) $x^3 - 4x + 15$ $f(-3)=0$ d) $x^3 + 4x + 15$ $f(-3)=-24$

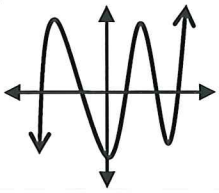
16. Match the following type of polynomials to the graphs.

A) Quadratic
B) Cubic
C) Quartic
D) Quintic

1)  B

2)  C

3) A 

4) D 

17. Which equation represents the graph of the function to the right?

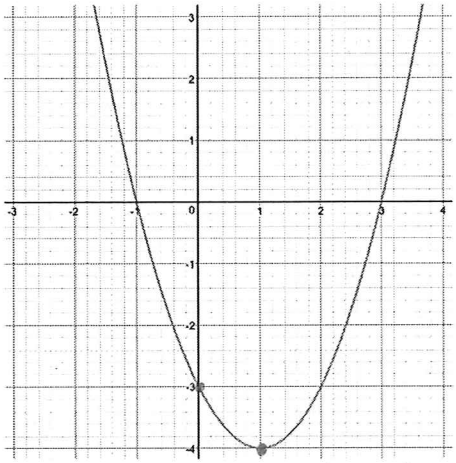
a. $y = x^2 - 3$
b. $y = x^2 + x + 4$
c. $y = x^2 - 2x - 3$
d. $y = x^2 - 4x + 1$

a) vertex $(0,-3)$ y-int $(0,-3)$

b) $\left(-\frac{1}{2}, \right)$

c) $\frac{2}{2} = 1$ $(1,-4)$

d) $\frac{4}{2} = 2$ $(2,-3)$



Vertex $(1,-4)$

18. Find a quadratic equation that has roots of 5 and -1?

$x = 5 \quad x = -1$

$(x-5)(x+1) =$ $x^2 - 4x - 5 = f(x)$

19. What is the maximum point of the graph of $y = -x^2 + 2x + 1$?

max $x = \frac{-b}{2a} = \frac{-2}{2(-1)} = 1$ $f(1) = -1 + 2 + 1 = 2$ $(1,2)$

20. What type of zeros does the function $f(x) = x^3 - x^2 + 9x - 9$ have?

2 imaginary
1 real

$x^2(x-1) + 9(x-1)$

$(x^2+9)(x-1)$

$x^2 + 9 = 0$ $x - 1 = 0$

$x^2 = -9$ $x = 1$

$x = \pm\sqrt{-9}$

$x = \pm 3i$

21. What are all the rational roots of $f(x) = x^3 + 2x - 6$?

Theorem! $5x^3 + 4x^2 + 1$ (q, p)

p: ± 1

q: $\pm 1 \pm 5$

p/q: $\pm 1 \pm \frac{1}{5}$ ★

	5	4	0	1
1	5	9	9	$10 \neq 0$
-1	5	-1	1	$0 = R$
1/5				
-1/5				

Quotient $5x^2 - 1x + 1 = 0$

$x = \frac{1 \pm \sqrt{(-1)^2 - 4(5)(1)}}{2(5)}$

$= \frac{1 \pm \sqrt{-19}}{10} = \frac{1 \pm i\sqrt{19}}{10}$

$x = -1$

22. Find the sum and product of the roots of the equation $2x^3 - 5x^2 + x - 3 = 0$.

a b constant

sum $= \frac{-b}{a} = \frac{5}{2}$ product $= \frac{\text{constant}}{a}$ (even degree) or $\frac{-\text{constant}}{a}$ (odd degree) $= \frac{3}{2}$

Solve the following inequalities #23 – 26.

23. $-2(x + 3) - 11 \geq 4$.

$-2x - 6 - 11 \geq 4$

$-2x - 17 \geq 4$

$+17$

$-2x \geq 21$

*flip ÷ neg

$x \leq -21/2$

24. $|4x - 1| < 2$

$4x - 1 < 2$ and $4x - 1 > -2$

$4x < 3$ $4x > -1$

$x < 3/4$ and $x > -\frac{1}{4}$

$-\frac{1}{4} < x < \frac{3}{4}$

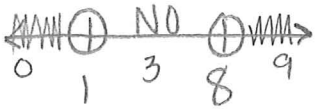
25. $|x + 2| \geq 5$

$x + 2 \geq 5$ OR $x + 2 \leq -5$

$x \geq 3$ OR $x \leq -7$

26. $x^2 - 9x + 8 > 0$

$(x - 8)(x - 1) > 0$



$x < 1$ OR $x > 8$

$f(0) = (-)(-) = + > 0$ True

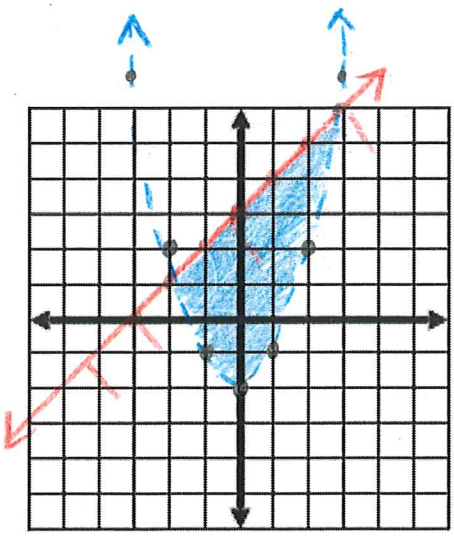
$f(3) = (-)(+) = - > 0$ False

$f(9) = (+)(+) = + > 0$ True

Graph the following inequalities.

27. $y > x^2 - 2$

$y \leq x + 3$

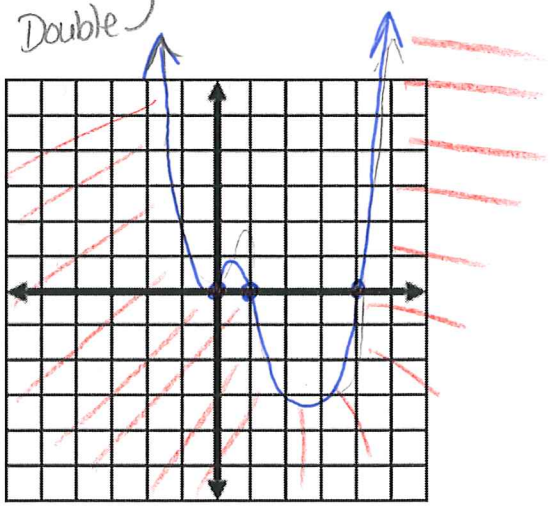


28. $y \leq x^2(x - 1)(x - 4)$

$+x^4$

$x=0$ $x=1$ $x=4$

Double



29. Find the zeros of $f(x) = x^3 + 7x^2 + 10x$.

$x(x^2+7x+10)$

$x(x+5)(x+2)$

$x=0$

$x=-5$

$x=-2$

30. Mrs. Hutschenreuter is hosting a cook-out. She is going to buy McDonald's burgers and fries. If consuming a 2000 calorie diet, the maximum daily requirement is 65 grams of fat and 70 grams of carbohydrates. Use the table to determine which inequalities describe how to obtain the maximum daily requirements.

	Fat	**Carbs**
Hamburger (x)	12 grams	31 grams
Small Fries (y)	11 grams	29 grams

a. $12x + 31y \le 65$
$11x + 29y \le 70$
$x \ge 0$
$y \ge 0$

b. $12x + 11y \le 65$
$31x + 29y \le 70$
$x \ge 0$
$y \ge 0$

c. $12x + 31y \ge 65$
$11x + 29y \ge 70$
$x \ge 0$
$y \ge 0$

d. $12x + 11y \ge 65$
$31x + 29y \ge 70$
$x \ge 0$
$y \ge 0$

31. State the domain of the function: $f(x) = \dfrac{x+2}{x^2-16}$.

Denominator $\neq 0$

$x^2 - 16 \neq 0$

$x^2 \neq 16$

$x \neq \pm\sqrt{16}$

$x \neq \pm 4$

$\{x \mid \mathbb{R}, x \neq \pm 4\}$

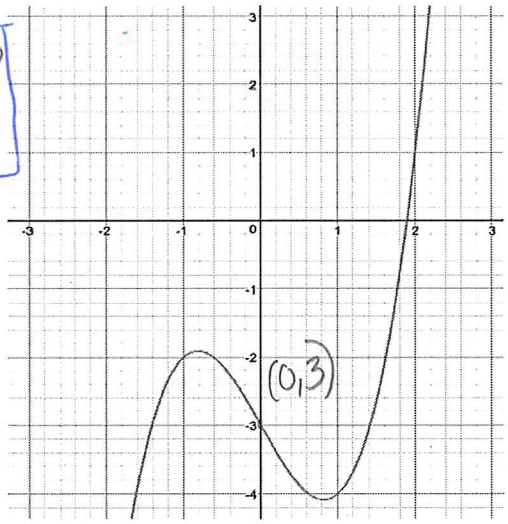
32. Which of the following could be a function for the graph to the right?

a. $y = x^3 - 3$ yint

b. $y = -x^3 + x + 4$

c. $y = x^3 - 2x - 3$ yint

d. $y = x^3 - 4x + 1$



$+x^3$

$-x^3$

If $f(x) = x^2 + 3$ and $g(x) = x - 4$, find the following compostions:

33. $[f \circ g](x)$?

$g(x) = x-4$

$f(x-4) = (x-4)^2 + 3$

$(x-4)(x-4) + 3$

$x^2 - 8x + 16 + 3$

$x^2 - 8x + 19$

34. $(f - g)(x)$

$(x^2+3) - (x-4)$

$x^2 + 3 - x + 4$

$x^2 - x + 7$

35. $(g \circ f)(5)$

$f(5) = (5)^2 + 3$

$25 + 3 = 28$

$g(28) = 28 - 4$

$= 24$

36. Does $y^2 - 4x^2 = 4$ have symmetry to the (a) x-axis, (b) y-axis, (c) y = x, and/or (d) the origin?

$y \to -y$ $x \to -x$ $x \leftrightarrow y$ $(-x, -y)$

a) $(-y)^2 - 4x^2 = 4$
$y^2 - 4x^2 = 4$ yes

b) $y^2 - 4(-x)^2 = 4 \to y^2 - 4x^2 = 4$ yes

c) $x^2 - 4y^2 = 4$ NO

d) $(-y)^2 - 4(-x)^2 = 4$
$y^2 - 4x^2 = 4$ yes

37. Match the function to the correct transformation?

M A. $3f(x) = 3 \cdot y$ — S. Shrink/compress Horizontally

S J. $f(3x) \to \frac{1}{3} \cdot x$ — L. Shrink/compress Vertically

L B. $\frac{1}{2} f(x) \to \frac{1}{2} \cdot y$ — K. Stretch Horizontally

K E. $f(\frac{1}{2} x) \to 2 \cdot x$ — M. Stretch Vertically

38. Given $f(x) = x^3 - 5$, find a rule for the inverse. (3 steps)

$y = x^3 - 5$ ① $x = y^3 - 5$ ② $x + 5 = y^3$

$y = \sqrt[3]{x+5}$ ③ $f^{-1}(x) = \sqrt[3]{x+5}$

39. **Multiple Choice:** The graph of $x^2 - y^2 = 9$ is symmetric with respect to:

a) x-axis ($y \to -y$) b) y-axis ($x \to -x$) c) neither **d) both axes**

40. Simplify $(-2x^2)^3(4x^{-4})$

$-2^3 x^6 \cdot 4x^{-4}$

$-8 x^6 \cdot 4 x^{-4}$

$-32 x^2$

41. Express $\frac{x^{\frac{4}{5}}}{x^{\frac{1}{5}}}$ using radicals.

$\frac{4}{5} - \frac{1}{5} = \frac{3}{5}$

$x^{3/5} \to \sqrt[5]{x^3}$

OR

$\left(\sqrt[5]{x}\right)^3$

42. Approximate to the nearest thousandths.

a) $\frac{14}{7} = \frac{7e^k}{7}$

$2 = e^k$

$\ln(2) = \ln e^k$

$\ln 2 = k \cdot \ln e$

$\ln 2 = k \cdot 1$

$k = \ln 2$

$k \approx .693$

b) $\frac{9}{4} = \frac{4e^{2x}}{4}$

$\frac{9}{4} = e^{2x}$

$\ln(9/4) = \ln e^{2x}$

$\frac{\ln 9/4}{2} = \frac{2x}{2}$

$x \approx .405$

43. Write $3^4 = 81$ in logarithmic form.

base

$\log_3(81) = 4$

44. Write $\log_{10} 0.01 = -2$ in exponential form

$10^{-2} = .01$

45. Simplify: $\log_4(3) + \log_4(3) - \frac{1}{2}\log_4(9)$

mult.

$\log_4 9 - \log_4 9^{1/2} = \log_4 9 - \log_4 \sqrt{9} = \log_4 9 - \log_4 3$

$= \log_4 \frac{9}{3} = \boxed{\log_4 3}$

46. Solve for x; round to the nearest hundredth.

$2^{2x} = 25$

*cannot change 25 to base 2 so...

$\log 2^{2x} = \log 25$

$2x \cdot \log 2 = \log 25 \rightarrow 2x = \frac{\log 25}{\log 2} \rightarrow \frac{2x}{2} = \frac{4.643}{2}$

$\boxed{x \approx 2.32}$

47. Solve $\frac{3}{4}\log_8 x = 2\log_8 2 + \log_8 2$

$\log_8 x^{3/4} = \log_8 4 + \log_8 2 \rightarrow \log_8 x^{3/4} = \log_8 8$

$x^{3/4} = 8 \rightarrow x = 8^{4/3} = (\sqrt[3]{8})^4 = 2^4$

*reciprocal

$\boxed{x = 16}$

48. **Matching**:

A)	Exponential Growth G	F)	$500(1.08)^6$
B)	Exponential Decay I	G)	5^x
C)	Continuous Growth H	H)	$4e^5$
D)	Growth model F	I)	$(\frac{1}{2})^x$
E)	Decay model J	J)	$500(.76)^4$

49. Find the natural logarithm of 7.

$\ln(7) \approx \boxed{1.946}$

50. Find the common logarithm of 7.

$\log(7) \approx \boxed{.845}$

51. Simplify: $\log_5 5^7$

$= \boxed{7}$ short cut!