Geometry: *Hutschenreuter* Name KEY

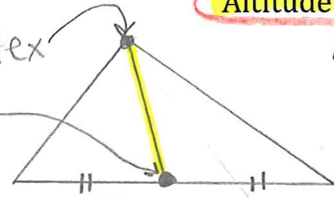
Section 5.4: Medians and Altitudes

Essential Question: What is the centroid and the orthocenter and how do you find them?

Vocabulary:

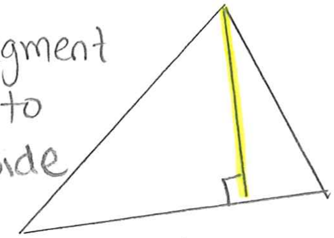
Median of a triangle

A segment from a vertex to the midpoint of the opposite side



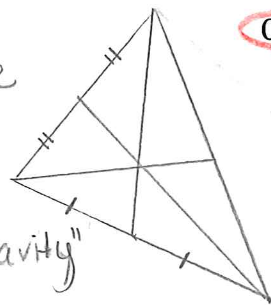
Altitude of a triangle

A perpendicular segment from a vertex to the opposite side



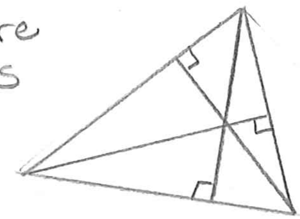
Centroid

The point where the 3 medians intersect
→ always inside △
→ A.K.A. "Center of Gravity"



Orthocenter

The point where the 3 altitudes intersect



Theorem 5.8: Concurrency of Medians of a Triangle Theorem

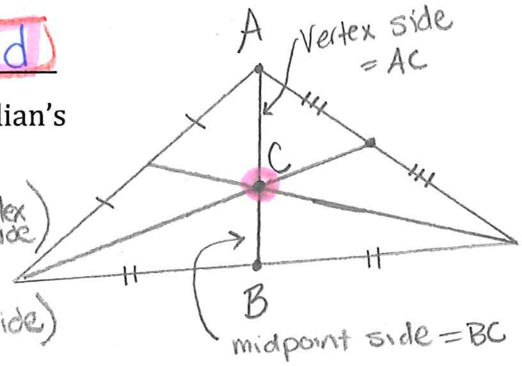
The medians of a triangle intersect at a point called the Centroid that divides each median into a "vertex-side" part that is 2/3 the median's length and a "midpoint-side" part that is 1/3 the median's length.

$\overline{AB}$ is a median

$\frac{\text{median}}{3}$ = midpt side

$BC = \frac{1}{2}(AC) \rightarrow$ Midpt side $= \frac{1}{2}$(vertex side)

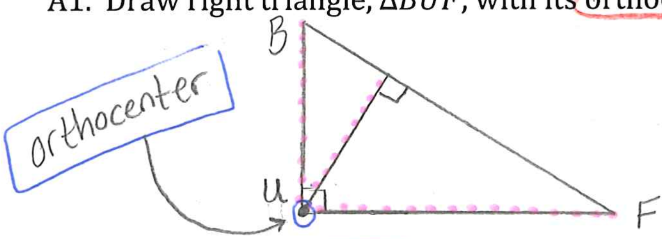
$AC = 2(BC) \rightarrow$ Vertex side = 2(midpt side)



Theorem 5.9: Concurrency of Altitudes of a Triangle Theorem

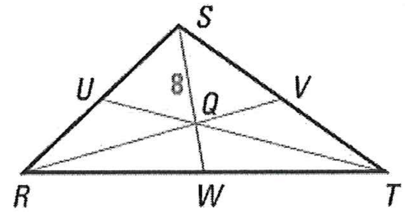
The lines containing the altitudes of a triangle intersect at a point called the Orthocenter. (See examples: #1 & 6)

A1. Draw right triangle, ΔBUF, with its orthocenter.



*All 3 altitudes intersect @ the rt. ∠ → the orthocenter of a rt. △ is always at the rt. ∠

A2. In ΔRST with centroid Q, find QW and SW.



Vertex side = SQ
Midpt side = QW

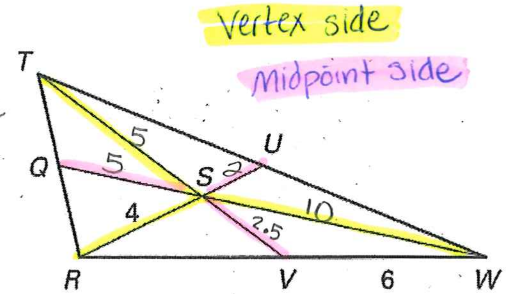
$SQ = 2(QW)$
$\frac{8}{2} = \frac{2(QW)}{2} \rightarrow \boxed{QW = 4\text{un}}$

$SW = SQ + QW$
$8 + 4$
$\boxed{SW = 12\text{un}}$

A3. In ΔRWT with medians $\overline{RU}, \overline{TV}$, and $\overline{WQ}$.
If $RS = 4$, $SV = 2.5$, and $QW = 15$, find...

QW ↳ median → $\frac{15}{3} = 5$ = midpt side
3 ← Thirds

a. $\boxed{SU = 2}$ b. $\boxed{TV = 7.5}$ c. $\boxed{SW = 10}$

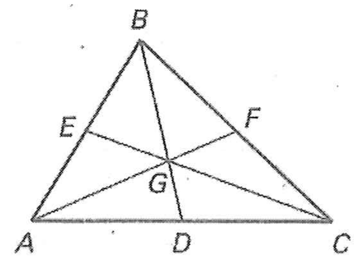


RS = vertex side
SU = Midpt side
$SU = \frac{1}{2}RS \rightarrow SU = \frac{1}{2}(4) = 2$

SV = 2.5 = midpt side
TS = 2(SV) = 2(2.5) = 5
TV = 2.5 + 5 = 7.5

SW = 2(SQ)
SW = 2(5) = 10

A4. Find the value of x if $CE = 6x$, $CG = 3x + 7$, and G is the triangle's centroid.



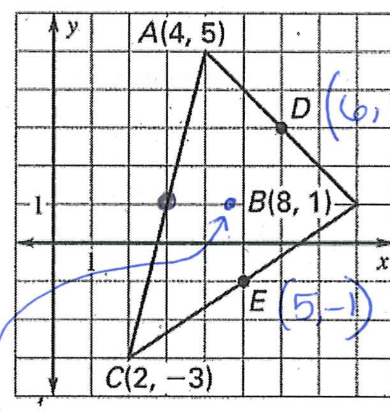
CG = 2(GE)
3x + 7 = 2(2x)
3x + 7 = 4x
−3x
7 = x

$\boxed{x = 7}$

CE = median
$\frac{CE}{3}$ = midpt. side = GE
$\frac{6x}{3}$ = GE
2x = GE

Remember... midpoint = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

A5. Use the gridded figure to find the coordinates of...



a. points D, E, and F (the midpoint of $\overline{AC}$)

use grid to find D & E

$D = \left(\frac{4+8}{2}, \frac{5+1}{2}\right) = \left(\frac{12}{2}, \frac{6}{2}\right) = D(6, 3)$

$E = \left(\frac{2+8}{2}, \frac{-3+1}{2}\right) = \left(\frac{10}{2}, \frac{-2}{2}\right) = E(5, -1)$

use formula → $F = \left(\frac{4+2}{2}, \frac{5+-3}{2}\right) = \left(6/2, 2/2\right) = F(3, 1)$

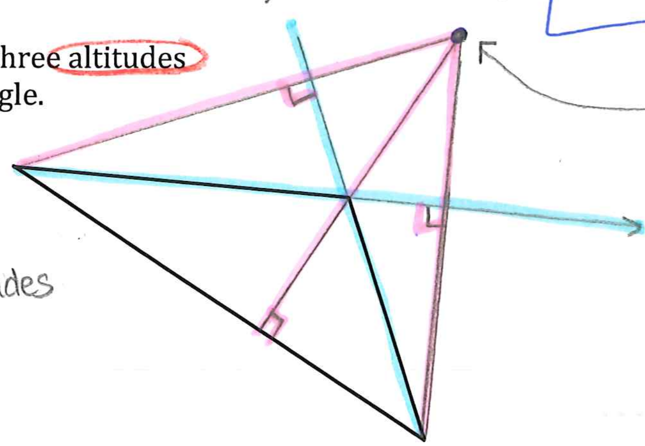
b. centroid = $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

$\left(\frac{4+8+2}{3}, \frac{5+1+-3}{3}\right) = \left(\frac{14}{3}, \frac{3}{3}\right) = \boxed{\left(4\frac{2}{3}, 1\right)}$

A6. Draw the three altitudes of the triangle.

① extend the Δ's sides

② Draw Altitudes



orthocenter

Section 5.4 Summary:

The centroid is the "center of gravity", which is the intersection of the 3 medians of a triangle. The centroid is always within Δ.

The orthocenter is the point of intersection of all three altitudes. There is no special characteristic of the orthocenter.