

**Section 6.1: Ratios, Proportions, and the Geometric Mean**

**Essential Question:** How do you use ratios and proportions to solve a problem?

**VOCABULARY:**

Ratio

The ratio of a to b is

$\frac{a}{b}$  or  $a:b$

Proportion

An equation stating that two ratios are equal

$\frac{a}{b} = \frac{c}{d}$   
 extreme → a, mean → c, mean → b, extreme → d

Geometric Mean

The x-value that makes

$\frac{a}{x} = \frac{x}{b}$  (equal)

is the geometric mean

**Cross Product (Means-Extremes) Property of Proportions:**

In a proportion, the product of the extremes equals the product of the means.

If  $\frac{a}{b} = \frac{c}{d}$ , then  $a \cdot d = b \cdot c$ .  
(cross multiply)

**Geometric Mean Property:**

The geometric mean of two positive numbers, a and b is the positive

number x that satisfies  $\frac{a}{x} = \frac{x}{b}$ , so  $x^2 = a \cdot b$  and  $x = \sqrt{a \cdot b}$ .

cross mult.

both sides

**EXAMPLES:**

**A1. Write the simplified ratio.**

a) The number of windows to doors for room 317 (this room).

w:d →  $2:1$

b) Rounded numbers to non-rounded numbers on the room clock.

(2, 3, 5, 6, 8, 9, 10, 12) (1, 4, 7, 11)

round = 8 # 's

4 # 's

8:4 reduce!

$2:1$

**A2. Simplify the ratio.**

a)  $\frac{30 \text{ yd}}{2} : \frac{22 \text{ yd}}{2}$

$15 \text{ yd} : 11 \text{ yd}$

\* equivalent ratios

b) 45 cm to 1m  
100cm

$\frac{45}{5} : \frac{100}{5}$

$9 \text{ cm} : 20 \text{ cm}$

c)  $\frac{4 \text{ ft}}{12 \text{ in}} = \frac{48 \text{ in}}{12 \text{ in}}$   
 $\div 12$   
 $\div 12$

$\frac{4 \text{ in}}{1 \text{ in}}$

A3. The ratio of boys to girl on the bus is 5:3. If there are 24 girls on the bus, how many boys are on the bus?

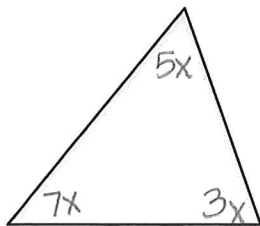
$$\frac{\text{boys}}{\text{girls}} = \frac{5}{3}$$

$$\frac{5}{3} \times \frac{x}{24}$$

$$3x = 5(24)$$

$$3x = 120$$

$$x = 40 \text{ boys}$$



A4. The extended ratio of the angles of a triangle is 7:5:3.

Find the measure of each angle.

$$7x + 5x + 3x = 180$$

$$15x = 180$$

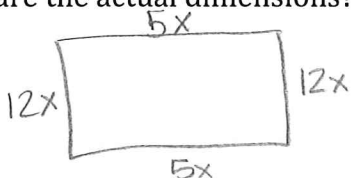
$$x = 12$$

$$3(12) = 36^\circ$$

$$5(12) = 60^\circ$$

$$7(12) = 84^\circ$$

A5. The length to width ratio of a mural is 5 to 12. If the perimeter of the Mural is 93.5 feet, what are the actual dimensions?



$$2(5x) + 2(12x) = 93.5$$

$$10x + 24x$$

$$\frac{34x}{34} = \frac{93.5}{34}$$

$$x = 2.75$$

(scale factor)

$$k = 2.75$$

$$5(2.75) = 13.75$$

$$\text{Width} = 13.75 \text{ ft}$$

$$12(2.75) = 33$$

$$\text{Length} = 33 \text{ ft}$$

A6. Solve the proportion.

a)  $\frac{4}{15} = \frac{x}{75}$

$$15x = 4(75)$$

$$15x = \frac{300}{15}$$

$$x = 20$$

b)  $\frac{y+3}{21} = \frac{y}{30}$

$$30(y+3) = 21y$$

$$30y + 90 = 21y$$

$$90 = -9y$$

$$\frac{90}{-9} = \frac{-9y}{-9}$$

$$y = -10$$

c)  $\frac{36}{x} = \frac{x}{9}$

$$x^2 = 36 \cdot 9$$

$$x^2 = 324$$

$$x = \pm \sqrt{324}$$

$$x = \pm 18$$

\* solving equation with  $\sqrt{\quad}$  need  $\pm$

A7. Find the geometric mean of the two numbers.

a) 20 and 5

only positive!

$$\frac{20}{x} = \frac{x}{5}$$

$$x^2 = 100 \quad x = \sqrt{100} = 10$$

b) 16 and 9

$$\frac{16}{x} = \frac{x}{9}$$

$$x^2 = 144 \quad x = \sqrt{144} = 12$$

c) 42 and 70

$$x^2 = 2940$$

$$x = \sqrt{2940} \quad x \approx 54.22$$

**Section 6.1 Summary:**

Set up a proportion of two items then cross multiply to solve for a missing mean or extreme.

$$\begin{array}{c} \text{extreme} \rightarrow \frac{a}{b} = \frac{c}{d} \leftarrow \text{mean} \\ \text{mean} \rightarrow \frac{a}{b} = \frac{c}{d} \leftarrow \text{extreme} \end{array}$$

$a \cdot d = b \cdot c$   
cross multiply, then divide.