KEY

Section 9.3: Law of Sines
Essential Question: When is the Law of Sines used?

Area of a triangle = ½ (a)(b) sin(C) = ½ (b)(c) sin(A) = ½ (a)(c) sin(B)

If you divide each by… ½ (a)(b)(c) you get: $\frac{\cancel{\frac{1}{2}ab}\sin C}{\cancel{\frac{1}{2}ab}c} = \frac{\cancel{\frac{1}{2}bc}\sin A}{\frac{1}{2}a\cancel{bc}} = \frac{\cancel{\frac{1}{2}ac}\sin B}{\cancel{\frac{1}{2}a}b\cancel{c}}$

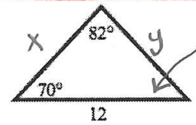
Law of Sines = $\boxed{\frac{\sin C}{c} = \frac{\sin B}{b} = \frac{\sin A}{a}}$

Step 1) set up a proportion (fraction = fraction)
Step 2) cross multiply
Step 3) divide (if solving for a side length this is your last step)

If solving for an angle one more step….
Step 4) take the sine inverse ($\sin^{-1}$)

When to use Law of Sines: $\boxed{ASA}$, $\boxed{AAS}$, $\boxed{SSA}$

Example 1
Solve the triangle.



$180-82-70=\boxed{28^\circ}$

$\frac{\sin 82^\circ}{12} = \frac{\sin 70^\circ}{y}$

$y = \frac{(12)(\sin 70)}{\sin 82}$

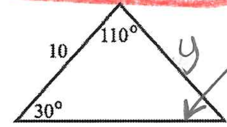
$\boxed{y = 5.69 \text{ un}}$

$\frac{\sin 82}{12} = \frac{\sin 28}{x}$

$x = \frac{(12)(\sin 28)}{\sin 82}$

$x = \boxed{11.39 \text{ un}}$

Example 2
Solve the triangle.



$180-110-30=\boxed{40^\circ}$

$\frac{\sin 40^\circ}{10} = \frac{\sin 30^\circ}{y}$

$y = \frac{(10)(\sin 30^\circ)}{\sin 40^\circ}$

$y = \boxed{7.78 \text{ un}}$

$\frac{\sin 40}{10} = \frac{\sin 110}{x}$

$x = \frac{(10)(\sin 110^\circ)}{\sin 40^\circ}$

$x = \boxed{14.62 \text{ un}}$

Test for number of possible triangles if given SSA:
Note: the value on the left side of the inequality is opposite of the given angle

1) a < bsinA then NO triangle exist

2) a = bsinA then 1 triangle exist

corresponding side & Angle

3) a > bsinA and a < b then 2 triangles exist

4) a > bsinA and a ≥ b then 1 triangle exist

corresponding side & angle

Example 3

Given the measurements of ΔXYZ, how many triangle(s) exist, if any?

$m\angle X = 30^{\circ}$ ← correspond

$x = 3$ ←

$y = 8$

$a \ \square \ b \cdot \sin A$

$3 \ \square \ 8 \sin 30^{\circ}$

$3 \ \boxed{<} \ 4$

less than → $\boxed{\text{NO } \Delta \text{ exist}}$

Also try...

$$\frac{\sin 30^{\circ}}{3} = \frac{\sin Y}{8}$$

$$\sin Y = \frac{(8)(\sin 30^{\circ})}{3}$$

$$\sin Y = 1.33$$

NO solution

↳ NO Δ

Cannot take $\sin^{-1}(1.33)$

Example 4

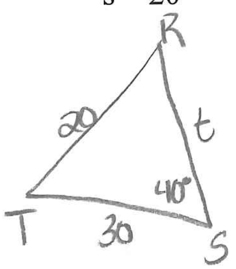
How many triangles exist? If any triangle(s) exist find all sides and angles.

ΔRST

$m\angle S = 40^{\circ}$

$r = 30$

$s = 20$



Need:
1) $\angle R$
2) $\angle T$
3) side t

$$\frac{\sin 40^{\circ}}{20} = \frac{\sin R}{30}$$

$$\sin R = \frac{(30)(\sin 40)}{20}$$

$$\sin R = .964$$

$$m\angle R = \sin^{-1}(.964)$$

$$\boxed{m\angle R = 74.6^{\circ}}$$

$$m\angle T = 180 - R - S$$
$$180 - 74.6 - 40$$

$$\boxed{m\angle T = 65.4^{\circ}}$$

$$\frac{\sin 65.4}{t} = \frac{\sin 40}{20} \qquad t = \frac{(20)(\sin 65.4)}{\sin 40^{\circ}}$$

$$\boxed{t = 28.29 \text{ un}}$$

2nd Δ

sin positive Q1 & Q2

$180 - 74.6 = \boxed{105.4 = m\angle R}$ ↑ yes 2nd Δ

$m\angle T = 180 - 105.4 - 40$

$$\boxed{m\angle T = 34.6^{\circ}}$$

$$\frac{\sin 34.6}{t} = \frac{\sin 40}{20}$$

$$t = \frac{(20)(\sin 34.6)}{\sin 40}$$

$$\boxed{t = 17.67 \text{ un}}$$

Section 9.3 Summary:

Law of Sines is used ASA and AAS and SSA

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

1) set up a proportion
2) cross multiply, then divide
3) Take the $\sin^{-1}(\ )$ if solving for an angle