Algebra III: *Hutschenreuter*
Chapters 7, 8, 9, and 17
SEMESTER II Review

Name KEY

Hour _____ Date _____

1. If $\sin\theta = \frac{-3}{5}$ and θ is a third quadrant angle, find the $\cos\theta$.

(-x, -y)
cos, sin

$\sin\theta = \frac{-3}{5} = \frac{y}{r}$

$x^2 + y^2 = r^2$

$y^2 + (-3)^2 = 5^2$

$x^2 = 16$

$x = \pm\sqrt{16}$

$x = -4$ (3)

$\cos\theta = \frac{x}{r} = \boxed{\frac{-4}{5}}$

2. If $\cos\theta = -1/3$ and $0 < \theta < 180^\circ$, find the $\sin\theta$.

$\cos\theta = \frac{-1}{3} = \frac{x}{r}$ $\quad x^2 + y^2 = r^2$ $\quad (-1)^2 + y^2 = 3^2$

$y^2 = 8$ $\quad y = \pm\sqrt{8}$

0° to 180° = pos

$\sin\theta = \frac{y}{r} = \frac{\sqrt{8}}{3} = \boxed{\frac{2\sqrt{2}}{3}}$

3. Find the exact value of each.

Look @ unit circle

a) $\cos\frac{\pi}{3}$ $\boxed{1/2}$

b) $\sin\frac{\pi}{4}$ $\boxed{\sqrt{2}/2}$

c) $\sin 225^\circ$ $\boxed{-\sqrt{2}/2}$

d) $\cos 135^\circ$ $\boxed{\frac{-\sqrt{2}}{2}}$

4. What is the value of …

a) $\cos^{-1}\left(\frac{1}{2}\right) = \boxed{60^\circ}$ or $\boxed{\frac{\pi}{3}}$

b) $\tan^{-1}(-1) = \boxed{-45^\circ}$ or $\boxed{\frac{-\pi}{4}}$

5. Express the value of each to four decimal places.

*MODE

a) $\sin 105^\circ$ $\quad .9659$

b) $\csc\frac{\pi}{4} = \frac{1}{\sin\frac{\pi}{4}}$ $\quad 1.4142$

c) $\sec 57^\circ = \frac{1}{\cos 57^\circ}$ $\quad 1.8361$

d) $\cot\frac{\pi}{3} = \frac{1}{\tan\pi/3}$ $\quad .5774$

6. Evaluate $\tan(\cos^{-1}\frac{1}{2})$.

$\cos\theta = \frac{1}{2} = \frac{x}{r}$ $\quad \tan\theta = \frac{y}{x}$

$1^2 + y^2 = 2^2$

$y^2 = 3 \rightarrow y = \sqrt{3}$

$\tan\theta = \frac{\sqrt{3}}{1} = \boxed{\sqrt{3}}$

7. A sector of a circle has a radius of 8 cm and a central angle of 27°. Find its arc length and area.

Arc length $= \frac{\theta}{360} \cdot 2\pi r$

$s = \frac{27}{360} \cdot 2\pi(8) \approx \boxed{3.77\text{ cm}} = s$

Area (Sector) $= \frac{\theta}{360} \cdot \pi r^2$

$A = \frac{27}{360} \cdot \pi(8)^2 \approx \boxed{15.08\text{cm}^2}$ = Area

8. Find two angles, one positive and one negative, which are coterminal with 60°.

$60^\circ + 360 = \boxed{420}, 780^\circ, \ldots$

$60^\circ - 360 = \boxed{-300^\circ}, -660^\circ$

↳ $\pm 360^\circ$ degree
↳ $\pm 2\pi$ radian

9. Simplify $(\tan x)\left(\frac{1}{\sec x}\right)$. $= \left(\frac{\sin}{\cos}\right)\left(\frac{\cos}{1}\right) = \sin x$

10. Express $\sin 400^\circ$ as a reference angle.

$400 - 360 = 40^\circ$

$\sin 40^\circ$

11. Simplify the following trigonometric expression.

$$\frac{1-\cos^2 x}{\sin^2 x+\cos^2 x} = \frac{\sin^2}{1} = \sin^2\theta$$

12. What is the amplitude and period of $y = -11 \sin \frac{1}{2} x$?

$Amp = |A| = |-11| = 11 = A$

$P = \frac{360}{B}$ or $\frac{2\pi}{B}$ $\quad P = \frac{360}{1/2} = 720^\circ = P$ or 4π

13. If $0 \le x \le 2\pi$, what are the solutions of: $\csc x = 2$

$\csc x = 2 \rightarrow \sin x = \frac{1}{2}$

look @ Unit Circle

$30^\circ, 150^\circ$

$\pi/6, 5\pi/6$

14. List the remaining four trigonometric identities in terms of their reciprocals. Two identities are already given.

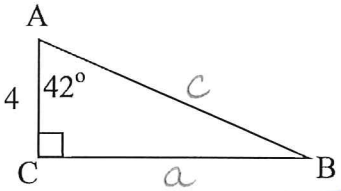
$$\tan\theta = \frac{1}{\cot\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$* \cot\theta = \frac{1}{\tan\theta}$ $\quad * \cos\theta = 1/\sec\theta$

$* \sin\theta = \frac{1}{\csc\theta}$

$* \csc\theta = 1/\sin\theta$

15. Solve the right triangle ABC.

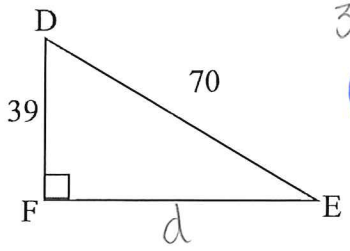


$180 - 90 - 42 = m\angle B = 48^\circ$

$\tan 42^\circ = \frac{a}{4}$ $\begin{matrix}\text{opp}\\ \text{adj}\end{matrix}$ $\quad a = 4 \cdot \tan 42^\circ = 3.6 \text{ un}$

$\frac{\cos 42^\circ}{1} = \frac{4}{c}$ $\quad c = 4(1) \div \cos 42^\circ = 5.38 \text{ un}$

16. Solve the right triangle DEF.



$39^2 + d^2 = 70^2$

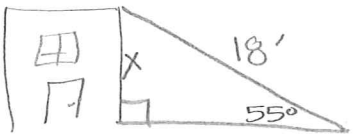
$d = 58.13 \text{ un}$

$\sin E = \frac{39}{70}$ $\begin{matrix}\text{opp}\\ \text{hyp}\end{matrix}$

$\sin^{-1}\left(\frac{39}{70}\right) = 33.86^\circ = m\angle E$

$180 - 90 - 33.86 = 56.14^\circ = m\angle D$

17. The safety instructions for a 18-foot ladder state that the ladder should not be at an incline greater than 55°. If the ladder was inclined at this angle, what is the height the ladder could reach?

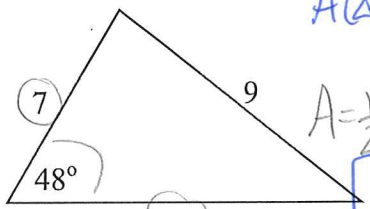


$\sin 55° = \frac{x}{18}$ $x = 14.74$ ft

Find the area of the triangles for problems #18 – #20.

$A(\Delta) = \frac{1}{2}ab \cdot \sin C$

18.

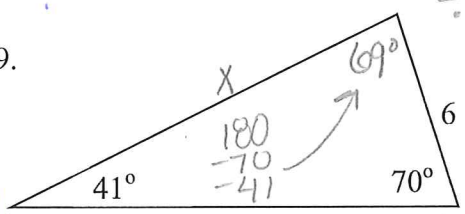


*SAS

$A = \frac{1}{2}(7)(10)\sin 48°$

$A = 26.01$ un²

19.



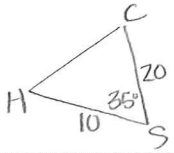
180 − 70 − 41

$\frac{\sin 41°}{6} = \frac{\sin 70°}{x}$ $x = 8.59$

$A = \frac{1}{2}(8.59)(6)\sin 69°$

$A = 24.07$ un²

20. In ΔCHS if c = 10, h = 20, and $m\angle S = 35°$.



*SAS

$A = \frac{1}{2}(10)(20)\sin 35° = 57.36$ un²

21. Solve triangle ABC if $\angle A = 40°$, $\angle B = 65°$, and a = 12.

*AAS → Law of Sines

All 3 ∠'s & sides

$\angle C = 75°$

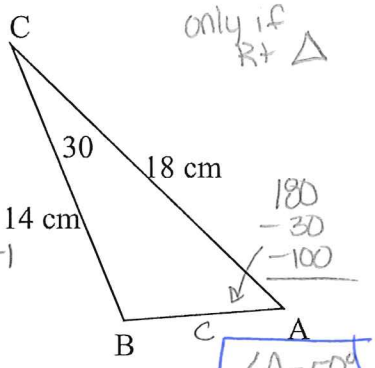
180 − 40 − 65 = 75

$\frac{\sin 40°}{12} = \frac{\sin 65}{b}$ $b = 16.92$

$\frac{\sin 40}{12} = \frac{\sin 75}{c}$ $c = 18.03$

* $a^2+b^2=c^2$ only if Rt △

22. Solve triangle ABC to the right.



*SAS → Law of Cosines

$c^2 = 14^2 + 18^2 - 2(14)(18)\cos 30°$

$\sqrt{c^2} = \sqrt{83.52}$ → $c = 9.14$ cm

$\cos B = \frac{18^2 - 14^2 - 9.14^2}{-2(14)(9.14)}$

$\cos B = -.1738$ → $\cos^{-1}$

180 − 30 − 100

$\angle B = 100°$

$\angle A = 50°$

23. If a = 10, b = 8, and c = 15, solve the triangle.

*SSS → Law of Cosines

$\cos A = \frac{10^2 - 8^2 - 15^2}{-2(8)(15)} = .7875$

$\cos^{-1}(.7875) = m\angle A = 38.05°$

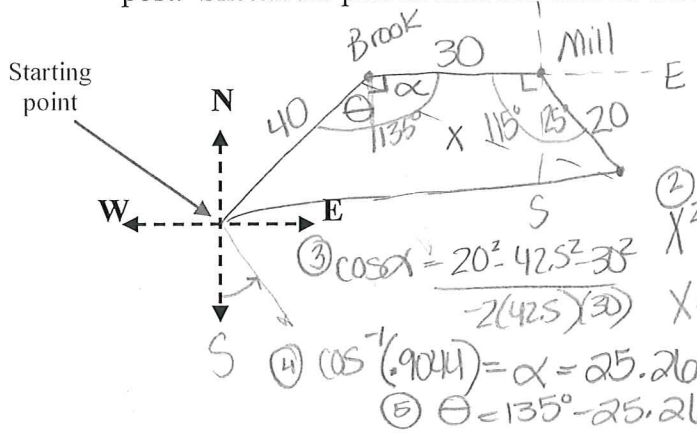
$\cos B = \frac{8^2 - 10^2 - 15^2}{-2(10)(15)} = .87$

$\cos^{-1}(.87) = m\angle B = 29.54°$

$\cos C = .38125$

$m\angle C = 112.41°$

24. From an iron post, proceed 40m NE to a brook, then 30m east along the brook to the old mill, then 200m S25°E to a post on the edge of Wiggin's Road and finally along Wiggin's Road back to the post. Sketch the plot of land and find its area.



$A = \frac{1}{2}ab\sin C$ 1st △

① $= \frac{1}{2}(30)(20)\sin 115° = 271.89$ m²

② $x^2 = 20^2 + 30^2 - 2(20)(30)\cos 115°$

$x = \sqrt{1807.14} = 42.5$

③ $\cos\alpha = \frac{20^2 - 42.5^2 - 30^2}{-2(42.5)(30)}$

④ $\cos^{-1}(.9044) = \alpha = 25.26$

⑤ $\theta = 135° - 25.26 = 109.74$

⑥ → 2nd △ $A = \frac{1}{2}(40)(30)\sin 109.74$

① + 564.74

Total = 836.63 m²

25. Charter High School has 1300 students and 100 staff members. The school newspaper interviewed 120 students and 20 teachers to see whether they were in favor of changing the schedule of the day to block scheduling. The results are listed in the table.

Stratum	Population Size	Sample Size	Number in favor of change
Students Staff	1300 100	120 20	62 9
Total	1400	140	71

Estimate the percent of the school in favor of block scheduling.

Students

$\left(\frac{1300}{1400}\right)\left(\frac{62}{120}\right) + \left(\frac{100}{1400}\right)\left(\frac{9}{20}\right) = .5119$ → 51.19%

Use the table below to answer #26 - #32.

Number of Students in Mrs. Hutsch's Classes for 3 school years								
School Year 12-13			School Year 13-14			School Year 14-15		
LQ	18	4th item		21	7th item	LE	14	1st item
	19	5th item		22	8th item		14	2nd item
	21	6th		25	9		17	3
	28	11th		29	14		26	10
	29	13th	UE	32	15	UQ	28	12

15 items

$\frac{15}{2} = 7.5$ → Med → 7 → 8th item

26. What is the mean?

$\frac{343}{15} = 22.87$

27. median? 22

28. mode? 14, 21, 28, 29 *multi-modal

29. What is the upper-quartile? 28

30. lower-quartile? 18

31. IQR? 28 − 18 = 10

32. What is the range of the data?

32 − 14 = 18

Use the following information to answer #33 and #34.

In a large school district, the set of all standardized math scores are normally distributed with a mean of 500 and standard deviation of 80.

33. What percent of scores are:

a) less than 450

$Z = \frac{450 - 500}{80} = -.6$

27.43%

b) greater than 575 (subtract from 100%)

$Z = \frac{575 - 500}{80} = .93$

.8159

100 − 81.59 = 18.41%

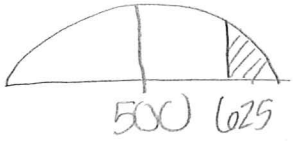
c) between 425 and 650 (subtract 2 percents)

$Z = -.93$ $Z = -.9$ 18.41%

$Z = 1.87$ $Z = 1.9$ 97.13%

97.13 − 18.41 = 78.72%

34. In a class of 30 students, how many would you expect to have a score greater than 625?



$Z = \frac{625 - 500}{80} = 1.56$ → .9452

100 − 94.52 = 5.48%

30(.0548) = 1.6

⇒ 1 or 2 students

35. Find the standard deviation of the set of test scores: 79, 98, 76, 84, 80, 77

1 2 3 4 5 6

$\bar{X} = \frac{494}{6} = 82.3$ σ^2 = variance $= \frac{(79-82.3)^2+(98-82.3)^2+\ldots+(77-82.3)^2}{6}$

$\sigma^2 = \frac{333.3}{6} = 55.556 \rightarrow \sigma = \sqrt{55.556}$

$\sigma = 7.45$

36. Find the five-number summary of the given data:

LE

2| 2 6 9 Med

LQ → 3|1 1 2 5 5

4| 0 8 9

5|1 3 4 7

UQ UE

3|1 = 31 years old

LE = 22 Med = 35 LQ = 31

UE = 57 UQ = 51

37. Math the function to the correct graph.

C 1. $f(x) = \sin x$ (0,0) a=1

A 2. $f(x) = -\frac{1}{2}\cos x$ (0, -1/2) a=

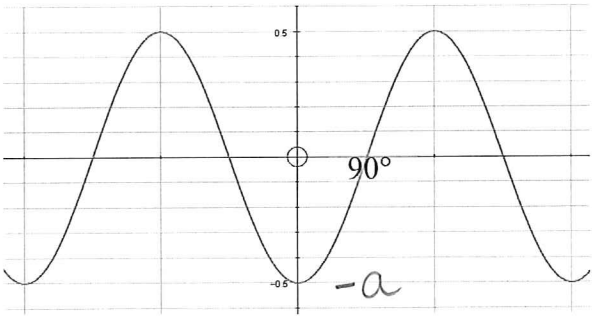
D 3. $f(x) = \cos(2x)$ (0, 1) a=1

B 4. $f(x) = \cos(\frac{1}{2}x)$ (0, 1)

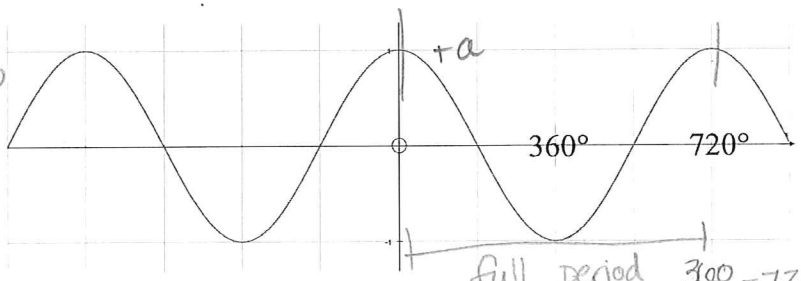
sin curve → (0,0)

cos curve → (0,a)

A) cos

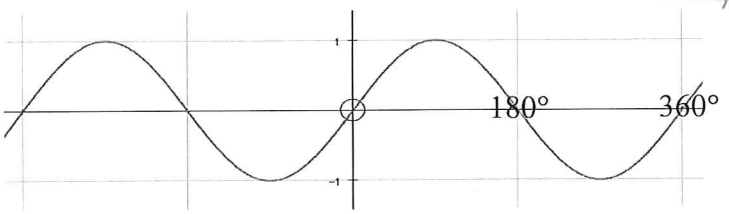


B) cos

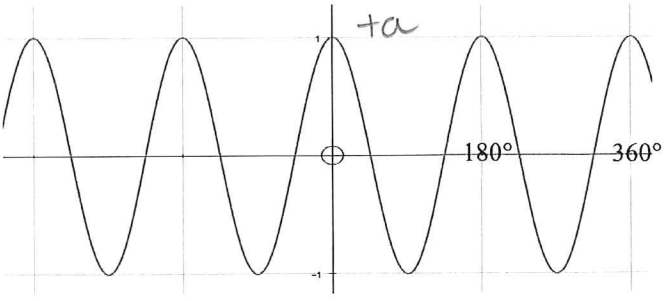


$\frac{360}{B} = 720°$ → B = 1/2

C) sin



D) cos



Multiple Choice.

38. If $0° \le x \le 360°$ solve $2\sin^2 x = 5\sin x + 3$

* substitute degrees into equation and figure which make a true solution

a) 30°, -30° b) 210°, 330° c) 30°, 330° d) 30°, 210°

$2\sin^2 x - 5\sin x - 3 = 0$

$2x^2 - 5x - 3 = 0$

$(2x+1)(x-3) = 0$

$2x+1=0$ $\quad x-3=0$

$x = -\frac{1}{2}$ $\quad x = 3$

$\sin\theta = -\frac{1}{2}$ → look on unit circle

$\theta = 330°$

$\theta = 210°$

$\sin\theta = 3$

$\emptyset$ outside of domain

Other ways to study:

- Look over chapter review packets and re-work the problems
- READ your chapter note cards = re-write them to one note sheet for the final exam
- Come to resource and look over past quizzes and test
- Attend MLT and study sessions

On the final exam, you may use:

1. Calculator
2. Note sheet – 1 full sheet of note book paper front/back (given to you)

No cell phones or music may be out during final exam time.
Come prepared:

1) Pencil
2) Eraser
3) Calculator
4) Water bottle
5) NOTE sheet!!