Geometry: *Hutschenreuter* Name KEY

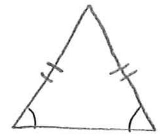
Section 7.4: Special Right Triangles

Essential Question:
How do you find the lengths of a 30° – 60° – 90° triangle and a 45° – 45° – 90° triangle?

Previous VOCABULARY:
Isosceles Triangle

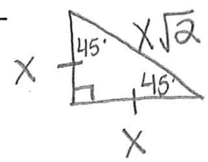
A triangle with two congruent sides

* Base angles are ≅



Theorem 7.8: 45° – 45° – 90° Triangle Theorem

In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.



$a^2+b^2=c^2$
$x^2+x^2=c^2$
$2x^2=c^2$

$\sqrt{c^2}=\sqrt{2x^2}$
$c=x\sqrt{2}$

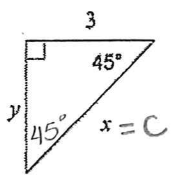
* hyp = (leg)($\sqrt{2}$)
* leg = $\frac{\text{hyp}}{\sqrt{2}}$

Note: has a $1:1:\sqrt{2}$ side ratio

EXAMPLES:

A1. Give the exact length of the missing sides.

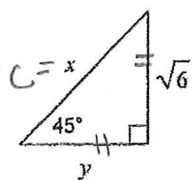
a.



$\boxed{y=3}$

x = hyp
x = leg($\sqrt{2}$)
$\boxed{x=3\sqrt{2}}$

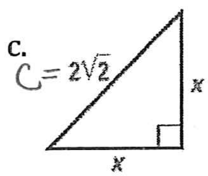
b.



$\boxed{y=\sqrt{6}}$

x = hyp
x = leg($\sqrt{2}$)
$x=\sqrt{6}\cdot\sqrt{2}$
$x=\sqrt{12}=\sqrt{4}\sqrt{3}=\boxed{2\sqrt{3}}$

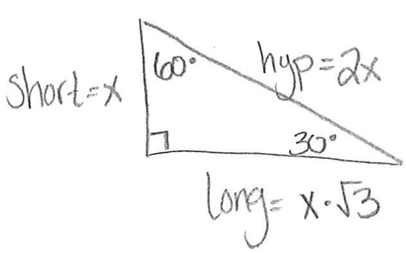
c.



leg = x
x = $\frac{\text{hyp}}{\sqrt{2}}$
x = $\frac{2\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}}$
$\boxed{x=2}$

Theorem 7.9: 30° – 60° – 90° Triangle Theorem

In a 30°-60°-90° triangle, the hypotenuse is 2 times as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

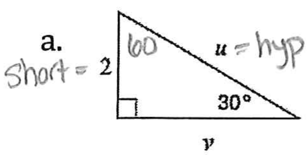


* hyp = 2(short)
* Long = (short)$\sqrt{3}$
* short = $\frac{\text{hyp}}{2}$
* Short = $\frac{\text{long}}{\sqrt{3}}$

Note: has a $1:2:\sqrt{3}$ side ratio

A2. Give the exact length of the missing sides.

a.



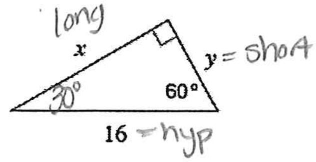
long = short $\sqrt{3}$

$\boxed{v = 2\sqrt{3}}$

hyp = 2(short)

$u = 2(2) \rightarrow \boxed{u = 4}$

b.



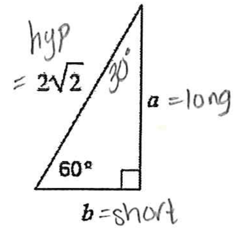
Short = $\frac{hyp}{2}$

$y = \frac{16}{2} \rightarrow \boxed{y = 8}$

long = (short)$\sqrt{3}$

$\boxed{x = 8\sqrt{3}}$

c.



short = $\frac{hyp}{2}$

$b = \frac{2\sqrt{2}}{2}$

$\boxed{b = \sqrt{2}}$

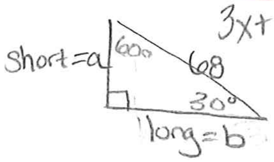
long = short $\sqrt{3}$

$a = \sqrt{2} \cdot \sqrt{3}$

$\boxed{a = \sqrt{6}}$

A3. Find the area of a triangle having a longest side of 68 cm with

a. An angle ratio of 3: 1: 2



$3x + 1x + 2x = 180$

$6x = 180$

$x = 30$

$3(30) = 90°$

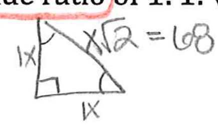
$1(30) = 30°$

$2(30) = 60°$

short = $\frac{hyp}{2}$ $\quad$ long = short$\sqrt{3}$ $\quad$ Area $= \frac{1}{2}(34)(34\sqrt{3})$

$a = \frac{68}{2} = 34$ $\quad$ $b = 34\sqrt{3}$ $\quad$ $\boxed{A = 1001.13 cm^2}$

b. A side ratio of 1: 1: $\sqrt{2}$



leg = $\frac{hyp}{\sqrt{2}}$

$x = \frac{68}{\sqrt{2}}$ (both legs)

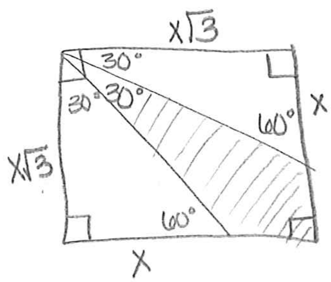
Area $= \frac{1}{2}\left(\frac{68}{\sqrt{2}}\right)\left(\frac{68}{\sqrt{2}}\right) = \frac{68 \cdot 68}{2 \cdot 2}$

$= \boxed{1156 cm^2}$

A4. Al baked a square pan of brownies for himself and his two sisters.
To divide the pan into three parts, he measured three 30° angles from one of the right-angled corners and cut the brownies into three large pieces. Two pieces form right triangles and the middle piece forms a kite.

a) Did Al divide the pan of brownies equally?

b) Explain your reasoning.



Area $\triangle$ (30-60-90) $= \frac{1}{2}$(Base)(Height) $= \frac{1}{2} x \cdot x\sqrt{3} \approx .866x^2$

$2\triangle$'s $= 2(.866x^2) = 1.732x^2$

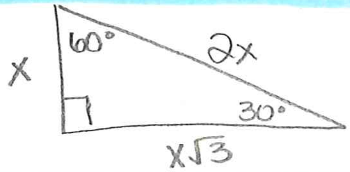
Area of $\square$ = (Length)(width) $= (x\sqrt{3})(x\sqrt{3}) = 3x^2$

Area of Kite $= \square - 2\triangle$'s $= 3x^2 - 1.732x^2 = 1.268x^2$

No, one $\triangle$ piece is $.866x^2$ and Kite piece is 1.268. $(1.268 > .866)$

Section 7.4 Summary:

In a 30-60-90° $\triangle$:

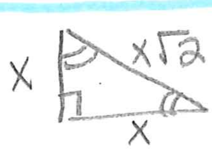


Short = $\frac{hyp}{2}$ = $\frac{long}{\sqrt{3}}$

hyp = 2(short)

long = short($\sqrt{3}$)

In a 45-45-90° $\triangle$:



hyp = leg($\sqrt{2}$)

leg = hyp/$\sqrt{2}$