Geometry: *Hutschenreuter* Name KEY

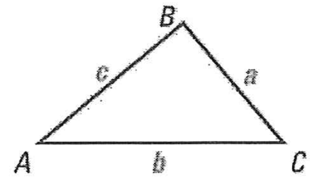
Section 7.8: Law of Sines and Law of Cosines

Essential Question:
In a non-right triangle, how can you find misssing sides and angles?

Key Concept: Law of Sines

In any triangle, the Law of Sines states that the ratio of the sine of an angle to its opposite side length is proportional throughout the triangle.

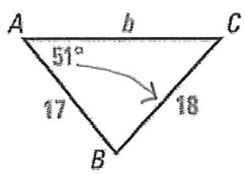
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



* Use when given
SSA
AAS
ASA

EXAMPLES:

A1. Find the value of *b*.



$\boxed{\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}}$

cannot start with:

$\dfrac{\sin 51°}{18} = \dfrac{\sin B}{b}$

two unknowns

* need ∠B to find side b

① $\dfrac{\sin 51°}{18} = \dfrac{\sin C}{17}$

② $\sin C = \dfrac{17(\sin 51°)}{18}$

③ $\sin C = .73397$

④ $\sin^{-1}(.73397)$

$m\angle C = 47.22°$

⑤ $\angle B$?

$180 - 51 - 47.22$

$m\angle B = 81.78°$

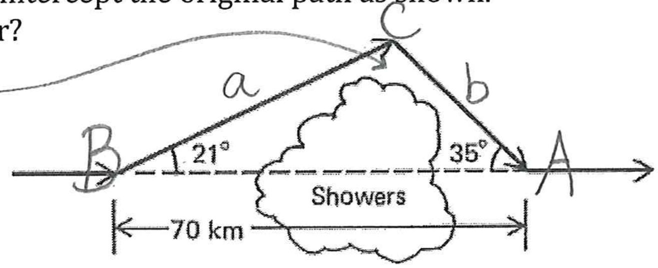
⑥ $\dfrac{\sin 51°}{18} = \dfrac{\sin 81.78}{b}$

⑦ $b = \dfrac{18(\sin 81.78)}{\sin 51°}$

⑧ $\boxed{b = 22.92\text{un}}$

A2. An airline pilot has to detour around a group of thunderclouds by turning left at an angle of 21° from the flight path, fly for a while, turn right, and intercept the original path as shown. How much futher did the pilot go due to the detour?

$m\angle C = 180 - 21 - 35 = 124°$



① $\dfrac{\sin 124}{70} = \dfrac{\sin 35°}{a}$

$a = \dfrac{70(\sin 35°)}{\sin 124°}$

② $a = 48.43$

③ $\dfrac{\sin 124}{70} = \dfrac{\sin 21°}{b}$

$b = \dfrac{70(\sin 21°)}{\sin 124}$

④ $b = 30.26$

⑤ $a + b = 48.43 + 30.26$

$a + b = 78.69$

How much more?

⑥ $78.69 - 70$

$= \boxed{8.69 \text{ km further}}$

A3. Find the measure of the largest angle of a triangle with its smallest angle of 17° and side lengths of 704 cm, 209 cm, and 637 cm.

Large small Medium

$$\frac{\sin 17°}{209} = \frac{\sin A}{704}$$

$$\sin A = \frac{704(\sin 17°)}{209}$$

$$\sin A = .9848$$

$$m\angle A = \sin^{-1}(.9848) = 80.01$$

$\boxed{\text{Largest Angle} = 80.01°}$

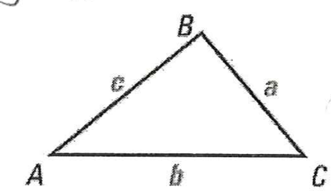
Key Concept: Law of Cosines

In any triangle, the Law of Cosines states that the square of a side is equal to the sum of the squares of the other side lengths less twice the product of the other side lengths and the cosine of the opposite angle.

To find a side length:

$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos C$

* Use when given SAS



To find an angle:

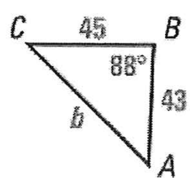
$$\cos C = \frac{c^2 - a^2 - b^2}{-2ab}$$

Corresponding

* Use when given SSS

EXAMPLES:

A4. Find the value of *b*.



$b^2 = a^2 + c^2 - 2ac \cdot \cos B$

$b^2 = 45^2 + 43^2 - 2(45)(43)\cos(88°)$

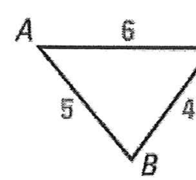
$b^2 = 3738.94$

$\sqrt{b^2} = \sqrt{3738.94}$

$\boxed{b \approx 61.15 \text{ un}}$

A5. Solve the triangle.

Find all Angles and sides



$\angle A$? $\angle B$? $\angle C$?

Have SSS use 2

$$\cos C = \frac{c^2 - a^2 - b^2}{-2ab}$$

side c

$$\cos C = \frac{5^2 - 4^2 - 6^2}{-2(4)(6)} = \frac{-27}{-48} = .5625$$

$m\angle C = \cos^{-1}(.5625) = \boxed{m\angle C \approx 55.77°}$

side a

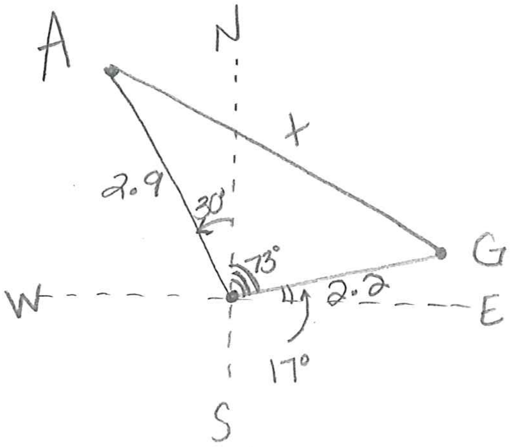
$$\cos A = \frac{4^2 - 5^2 - 6^2}{-2(5)(6)} = \frac{-45}{-60} .75$$

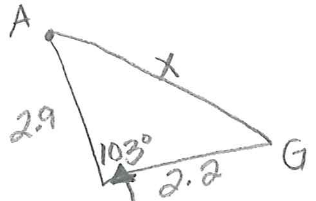
$m\angle A = \cos^{-1}(.75) = \boxed{m\angle A \approx 41.41°}$

$m\angle B = 180 - \angle A - \angle C = \boxed{m\angle B \approx 82.82°}$

to to (KEY)

A6. Greg and Alice park their car and start hiking. Greg heads 17° North of East and goes 2.2 miles, while Alice heads 30° West of North for 2.9 miles. How far apart are Greg and Alice?





$$\begin{array}{r} 73 \\ +30 \\ \hline 103 \end{array}$$

$$x^2 = 2.9^2 + 2.2^2 - 2(2.9)(2.2)\cos 103^\circ$$

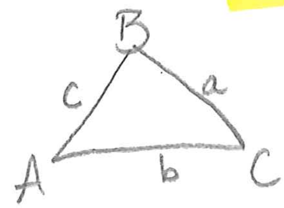
$$\sqrt{x^2} = \sqrt{16.12}$$

$$x \approx \boxed{4.02 \text{ miles apart}}$$

Summary 7.8:

* If given SSA, AAS, ASA use the Law of Sines:

$$\boxed{\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}}$$



to find a side or angle.

1. set up proportion
2. cross multiply
3. Divide
4. $\sin^{-1}$ if solving for an angle

* If given SSS you can find all missing angles using Law of Cosines:

$$\boxed{\cos C = \frac{c^2 - a^2 - b^2}{-2(a)(b)}}$$

correspond ∠ and side

1. solve for $\cos C$
2. take the $\cos^{-1}$

* If give SAS you can find the missing side using Law of Cosines:

$$\boxed{c^2 = a^2 + b^2 - 2(a)(b)\cos C}$$

side corresponds to angle

1. solve for c^2
2. take the square root $\sqrt{\ }$