Geometry: *Hutschenreuter* Name KEY
Chapter 8 Note Packet

* Look @ p43 in text for polygon names

Section 8.1: Find Angle Measures in Polygons

Essential Question:
How do you find a missing angle measure in a convex polygon?

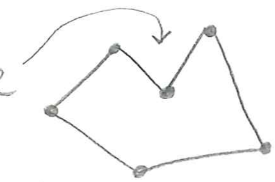
VOCABULARY:

Polygon
- Has 3 or more sides
- Each side intersects exactly 2 sides

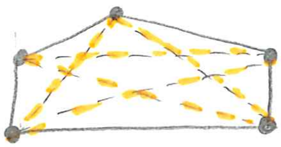
Regular Polygon
A polygon that has all ≅ sides and all angles are ≅

Convex Polygon
Not "pushed" in

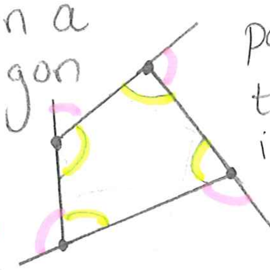
Concave Polygon
- "pushed" in angle
- collapsed



Diagonal
A segment that joins two nonconsecutive vertices



Interior angle
An angle within a polygon



Exterior angle
When the sides of a polygon are extended, the ∠ adjacent to the interior angle

Theorem 8.1: Polygon Interior Angles Theorem

The sum of the measures of the interior angles of an n-sided convex polygon is $(n-2)\cdot 180$.

sum of int ∠'s $= (n-2)(18)$

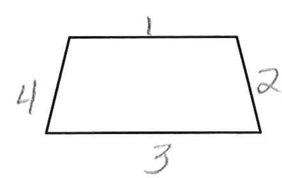
ex:  Pentagon → n = 5 sides ∴ sum $= (5-2)(180)$
$(3)(180)$
Sum $= 540°$

A1. Find the interior angle sum of the convex polygon.

a. 14-gon

sum of int ∠ $= (n-2)(180)$
sum $= (14-2)(180)$
$= (12)(180)$
$\boxed{\text{sum} = 2160°}$

b.



n = 4

sum $= (4-2)(180)$
$(2)(180)$
$\boxed{\text{sum} = 360°}$

Corollary to Theorem 8.1: Quadrilateral Angle Sum

The sum of the measures of the interior angles of a quadrilateral is $360°$.
↳ 4 sides, n = 4

sum $= (4-2)(180) = 2(180) = 360°$
$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360°$

3 = Triangle = 180° 5 = Pentagon = 540° 7 = Heptagon = 900° 9 = Nonagon = 1260°
4 = Quadrilateral = 360° 6 = Hexagon = 720° 8 = Octagon = 1080° 10 = Decagon = 1440°

A2. Explain why…

a) All quadrilaterals have an interior angle sum of 360°

Quadrilaterals have 4 sides, therefore $n=4$ which gives us sum $= (4-2)(180) = 2(180) = 360°$

b) All triangles have an interior angle sum of 180°

All Δ's have 3 sides, therefore $n=3$ which gives us sum $(3-2)(180) = 1(180) = 180°$

A3. Use the interior angle measure sum to classify the convex polygon.

a. 900° = sum on int. $\angle$s

$900 = (n-2)(180)$

$\frac{900}{180} = \frac{(n-2)(\cancel{180})}{\cancel{180}}$

$5 = n-2$

$+2 \quad +2$

$\boxed{n=7 \text{ Heptagon}}$

b. 1800°

$(n-2)(180) = 1800$

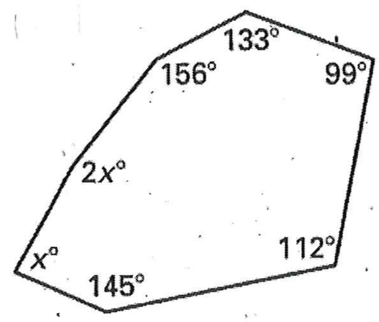
$\frac{(n-2)(\cancel{180})}{\cancel{180}} = \frac{1800}{180}$

$n-2 = 10$

$+2 \quad +2$

$\boxed{n=12 \text{ Dodecagon}}$

A4. Find the value of x.



7 sides → $n=7$

$(n-2)(180) = (7-2)180$

$= (5)(180)$

$= 900 = $ sum of $\angle$'s

$x + 2x + 156 + 133 + 99 + 112 + 145 = 900$

$3x + 645 = 900$

-645

$\frac{3x}{3} = \frac{255}{3}$

$\boxed{x = 85}$

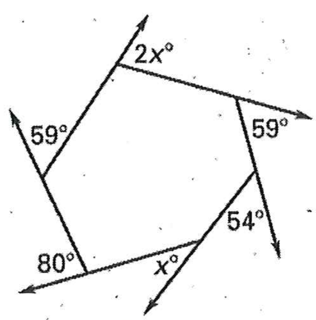
Theorem 8.2: Polygon Exterior Angles Theorem

The sum of the measures of the __exterior__ angles of a __convex__ polygon, one angle at each vertex, is __360°__.

Given n-sided polygon, the exterior angles add up to 360°

$m\angle 1 + m\angle 2 + m\angle 3 + \ldots + m\angle n = 360°$

A5. Find the value of x.



All are exterior angles

$2x + 59 + 54 + x + 80 + 59 = 360$

$3x + 252 = 360$

-252

$\frac{3x}{3} = \frac{108}{3}$

$\boxed{x = 36}$

- Interior Angle Measure Sum (Convex Polygon): $(n-2)180°$ (n = # of sides)
- Exterior Angle Measure Sum (Convex Polygon): 360°
- Measure of one interior angle = $\frac{(n-2)180}{n}$ Measure of one exterior angle = $\frac{360}{n}$

A6. Find the measure of an interior angle and an exterior angle of the given regular polygon.

a. Regular heptagon $\rightarrow n=7$

$\text{Int}\angle = \frac{(7-2)(180)}{7} = \frac{(5)(180)}{7} = \frac{900}{7} = \boxed{128.57°}$

$\text{Ext}\angle = \frac{360}{7} = \boxed{51.43°}$

$128.57+51.43=180$ ✓

b. Regular nonagon $\rightarrow n=9$

$\text{Int}\angle = \frac{(9-2)(180)}{9} = \frac{7(180)}{9} = \boxed{140°}$

$\text{Ext}\angle = \frac{360}{9} = \boxed{40°}$

$140 + 40 = 180$ ✓

* ***Note: The sum of an adjacent interior and exterior angle is*** 180°.

A7. Classify the polygon with the given information.

a) Each exterior angle is 10°.

$\text{Ext}\angle = \frac{360}{n}$

$\frac{10}{1} = \frac{360}{n} \rightarrow n = \frac{1(360)}{10}$

$\boxed{n=36}$

$\boxed{36\text{-gon}}$

b) Each interior angle is 108°.

$\text{Int} + \text{Ext} = 180$

$108 + \text{Ext} = 180$

-108

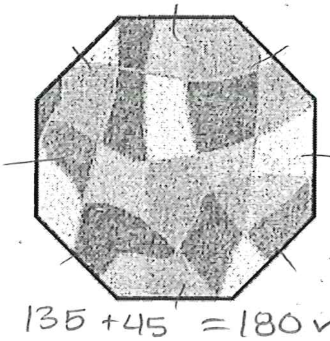
$\text{Ext}\angle = 72°$

$\frac{72°}{1} = \frac{360}{n} \rightarrow n = \frac{1(360)}{72} = 5$

$\boxed{n=5}$

$\boxed{\text{Pentagon}}$

A8. The regular polygon is a stained glass window.
Find the measures of an interior angle and an exterior angle.



$n=8$

$\text{Int}\angle = \frac{(8-2)(180)}{8} = \frac{6(180)}{8} = \frac{1080}{8} = \boxed{135° = \text{Int}\angle}$

$\text{Ext}\angle = \frac{360}{n} = \frac{360}{8} = \boxed{45° = \text{Ext}\angle}$

Check: $135+45 = 180$ ✓

A9. The measures of the interior angles of a convex pentagon are $3x°$, $4x°$, $5x°$, $7x°$, and $8x°$.
Find the largest exterior angle measure. $\rightarrow n=5$

① $\text{Int}\angle \text{ sum} = (n-2)(180)$
$= (5-2)(180)$
$3(180)$
$\text{Sum} = 540$

② $3x+4x+5x+7x+8x = 540$
$\frac{27x}{27} = \frac{540}{27}$

③ $x = 20$

④ small int $\angle = 3x = 3(20) = 60$

⑤ $180-60 = \boxed{120°}$
$\boxed{\text{Largest ext}\angle}$

* Largest ext will pair with smallest int $\angle$

Summary 8.1:

→ To find an angle in a regular polygon calculate the sum of the interior angles the divide by the # of angles

An int. $\angle = \frac{(n-2)(180)}{n}$

→ To find one missing angle in a nonregular polygon subtract all given angles from the sum of $(n-2)(180)$